
بسم الله الرحمن الرحيم

Applied Thermodynamics

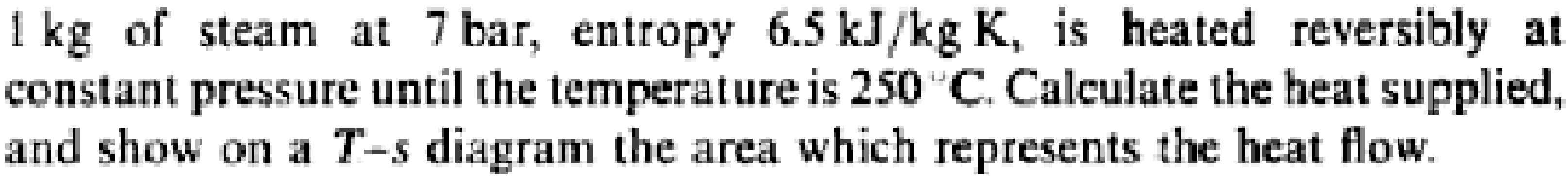
Week_13

Instructor: Mr. Adnan Qamar

Mechanical Engineering Department

ENTROPY

Example 4.1:



1 kg of steam at 7 bar, entropy 6.5 kJ/kg K, is heated reversibly at constant pressure until the temperature is 250 °C. Calculate the heat supplied, and show on a T–s diagram the area which represents the heat flow.

Given Data:

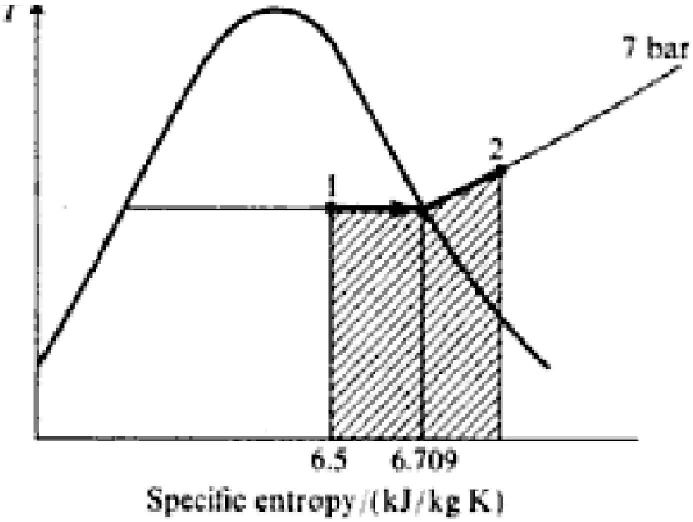
m = 1 kg   T1 = 650°C   s1 = 6.5kJ/kg.K   P1=P2   T2=250°C

Q=?   T-s diagram?

ENTROPY

Solution:

At 7 bar, $s_g = 6.709$ kJ/kg K, hence the steam is wet, since the actual entropy, s_1 is less than s_g.



From equation (4.12)

$$x_1 = \frac{s_1 - s_{f1}}{s_{fg1}} = \frac{6.5 - 1.992}{4.717} = 0.955$$

Then from equation (2.2)

$$h_1 = h_{f1} + x_1 h_{fg1} = 697 + (0.955 \times 2067)$$

i.e. $h_1 = 697 + 1975 = 2672$ kJ/kg

At state 2 the steam is at 250°C at 7 bar, and is therefore superheated. From superheat tables, $h_2 = 2955$ kJ/kg.

At constant pressure from equation (3.3)

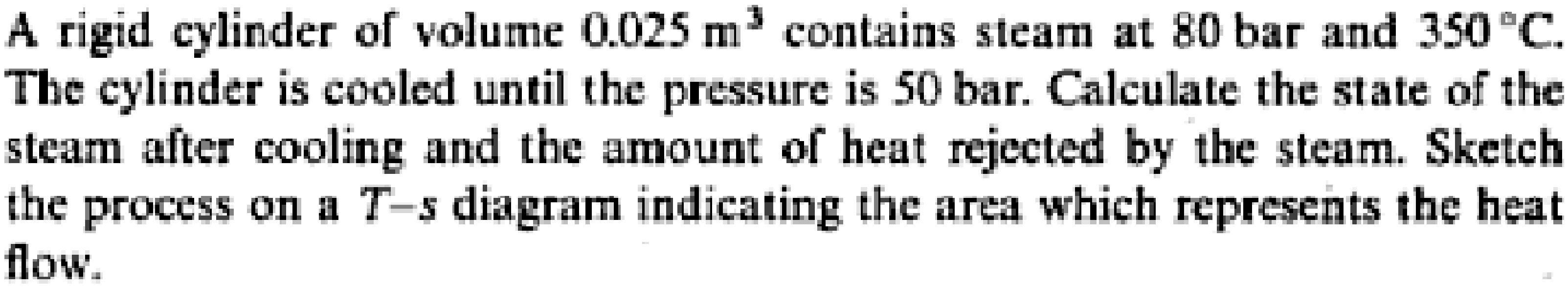
$$Q = h_2 - h_1 = 2955 - 2672 = 283 \text{ kJ/kg}$$

i.e. Heat supplied = 283 kJ/kg

The T–s diagram showing the process is given in Fig. 4.9, the shaded area representing the heat flow.

ENTROPY

Example 4.2:



A rigid cylinder of volume 0.025 m^3 contains steam at 80 bar and 350 °C. The cylinder is cooled until the pressure is 50 bar. Calculate the state of the steam after cooling and the amount of heat rejected by the steam. Sketch the process on a T–s diagram indicating the area which represents the heat flow.

Given Data:

V1=V2=0.025m3 P1=80 bar T1=350°C

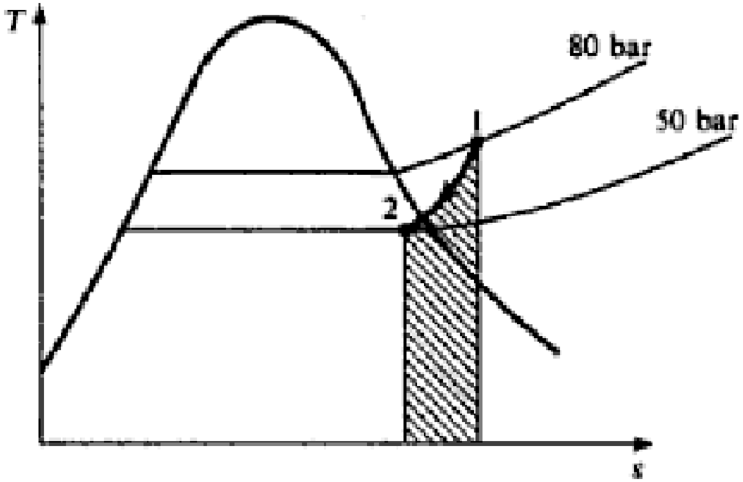
P2=50 bar x2=? Q=? T-s diagram?

ENTROPY

Solution:

Steam at 80 bar and 350 °C is superheated, and the specific volume from tables is 0.029 94 m^3/kg. Hence the mass of steam in the cylinder is given by

$$m = \frac{0.025}{0.029\,94} = 0.835\ \text{kg}$$



For superheated steam above 80 bar the internal energy is found from equation (3.7),

$$u_1 = h_1 - p_1 v_1 = 2990 - \frac{80 \times 10^5 \times 0.02994}{10^3} = 2990 - 239.5$$

i.e. $u_1 = 2750.5\ \text{kJ/kg}$

ENTROPY

At state 2, $p_2 = 50$ bar and $v_2 = 0.029\,94\ \mathrm{m^3/kg}$, therefore the steam is wet, and the dryness fraction is given by equation (2.1)

$$x_2 = \frac{v_2}{v_{g_2}} = \frac{0.029\,94}{0.039\,44} = 0.758$$

From equation (2.3)

$$u_2 = (1 - x_2)u_{f_2} + x_2 u_{g_2} = (0.242 \times 1149) + (0.758 \times 2597)$$

i.e. $u_2 = 278 + 1969 = 2247\ \mathrm{kJ/kg}$

At constant volume from equation (3.2),

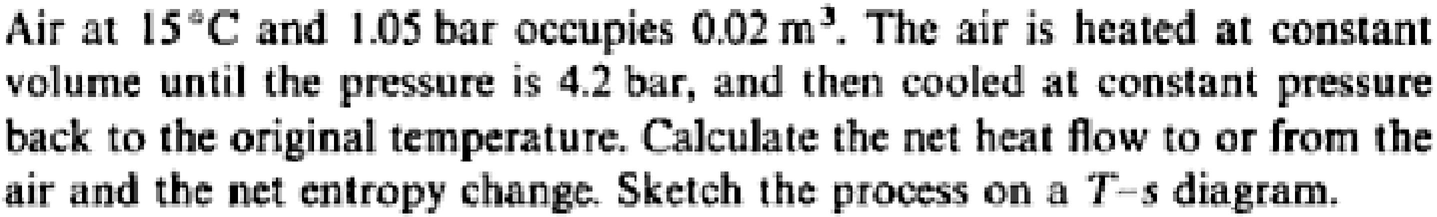
$$Q = U_2 - U_1 = m(u_2 - u_1) = 0.835(2247 - 2750.5)$$

i.e. $Q = -0.835 \times 503.5 = -420\ \mathrm{kJ}$

i.e. Heat rejected = 420 kJ

ENTROPY

Example 4.3:



Air at 15 °C and 1.05 bar occupies $0.02\ m^3$. The air is heated at constant volume until the pressure is 4.2 bar, and then cooled at constant pressure back to the original temperature. Calculate the net heat flow to or from the air and the net entropy change. Sketch the process on a *T–s* diagram.

Given Data:

T1=15°C P1=1.02 bar V1=0.02m3

V2=V1 P2=4.2 bar P3=P2

T3=T1 Q=? Δs=?

T-s diagram?

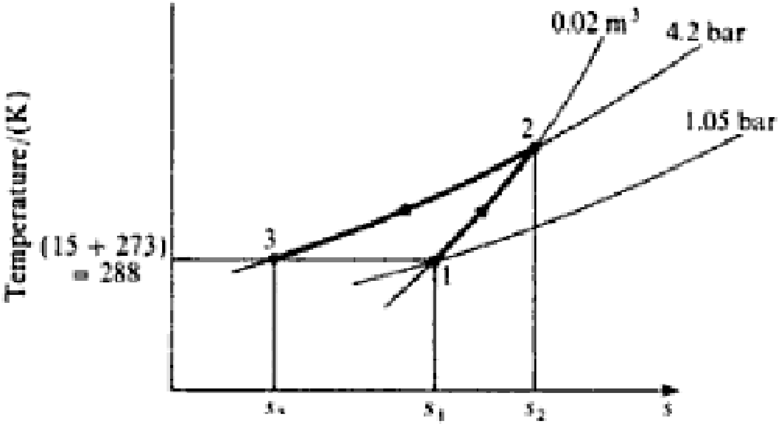
ENTROPY

Solution:

The processes are shown on a T–s diagram in Fig. 4.13. From equation (2.6), for a perfect gas,

$$m = \frac{pV}{RT} = \frac{1.05 \times 10^5 \times 0.02}{0.287 \times 10^3 \times 288} = 0.0254 \text{ kg}$$

where $T_1 = 15 + 273 = 288$ K.



For a perfect gas at constant volume, $p_1/T_1 = p_2/T_2$, hence

$$T_2 = \frac{4.2 \times 288}{1.05} = 1152 \text{ K}$$

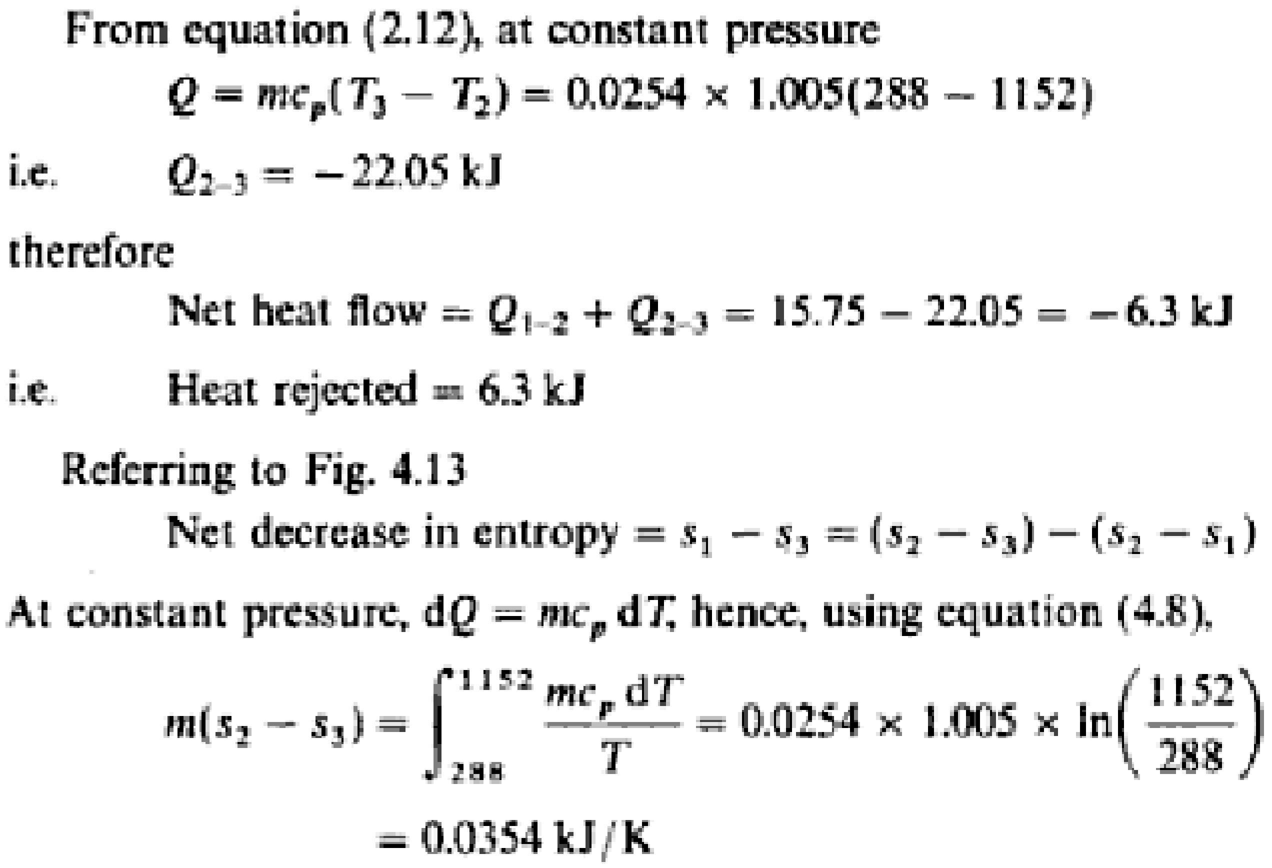
From equation (2.13), at constant volume

$$Q = mc_v(T_2 - T_1) = 0.0254 \times 0.718(1152 - 288)$$

i.e. $Q_{1\text{-}2} = 15.75$ kJ

ENTROPY

Solution:



From equation (2.12), at constant pressure

$$Q = mc_p(T_3 - T_2) = 0.0254 \times 1.005(288 - 1152)$$

i.e. $Q_{2-3} = -22.05$ kJ

therefore

$$\text{Net heat flow} = Q_{1-2} + Q_{2-3} = 15.75 - 22.05 = -6.3 \text{ kJ}$$

i.e. Heat rejected = 6.3 kJ

Referring to Fig. 4.13

$$\text{Net decrease in entropy} = s_1 - s_3 = (s_2 - s_3) - (s_2 - s_1)$$

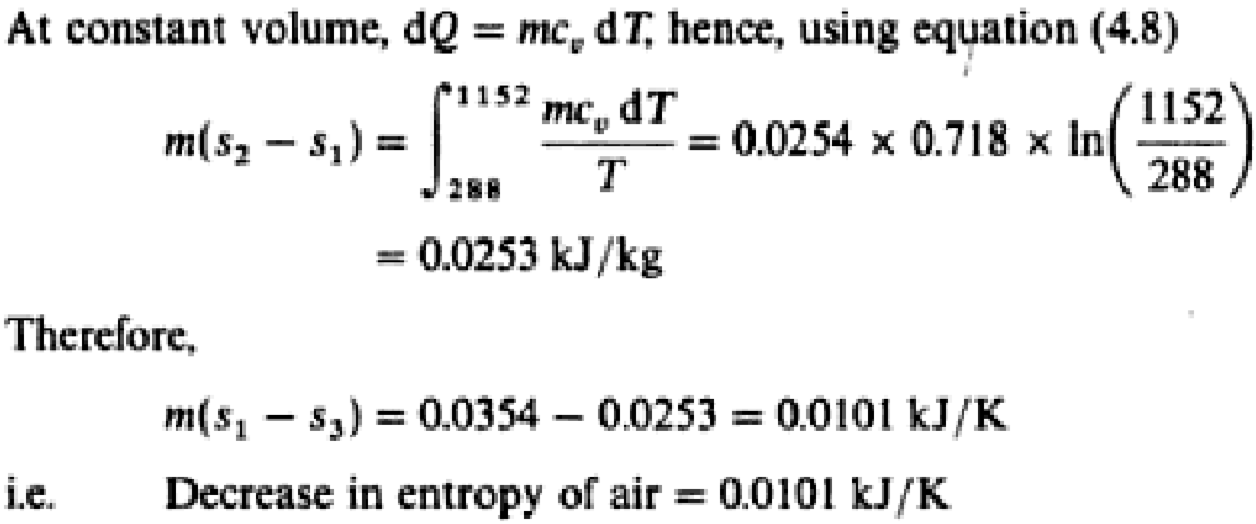
At constant pressure, $dQ = mc_p\,dT$, hence, using equation (4.8),

$$m(s_2 - s_3) = \int_{288}^{1152} \frac{mc_p\,dT}{T} = 0.0254 \times 1.005 \times \ln\left(\frac{1152}{288}\right)$$

$$= 0.0354 \text{ kJ/K}$$

ENTROPY

Solution:



At constant volume, $dQ = mc_v\,dT$, hence, using equation (4.8)

$$m(s_2 - s_1) = \int_{288}^{1152} \frac{mc_v\,dT}{T} = 0.0254 \times 0.718 \times \ln\left(\frac{1152}{288}\right)$$

$$= 0.0253 \text{ kJ/kg}$$

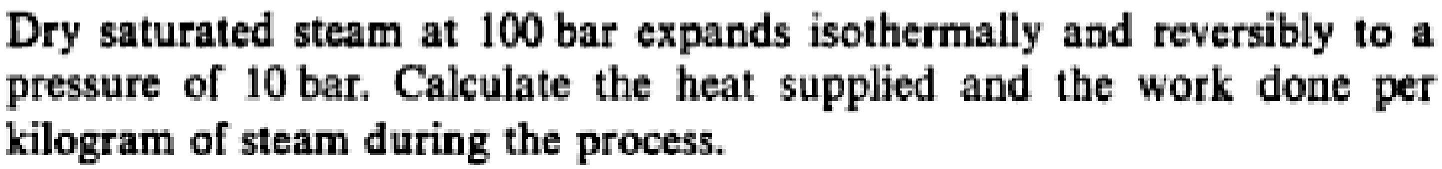
Therefore,

$$m(s_1 - s_3) = 0.0354 - 0.0253 = 0.0101 \text{ kJ/K}$$

i.e. Decrease in entropy of air = 0.0101 kJ/K

ENTROPY

Example 4.4:



Dry saturated steam at 100 bar expands isothermally and reversibly to a pressure of 10 bar. Calculate the heat supplied and the work done per kilogram of steam during the process.

Given Data:

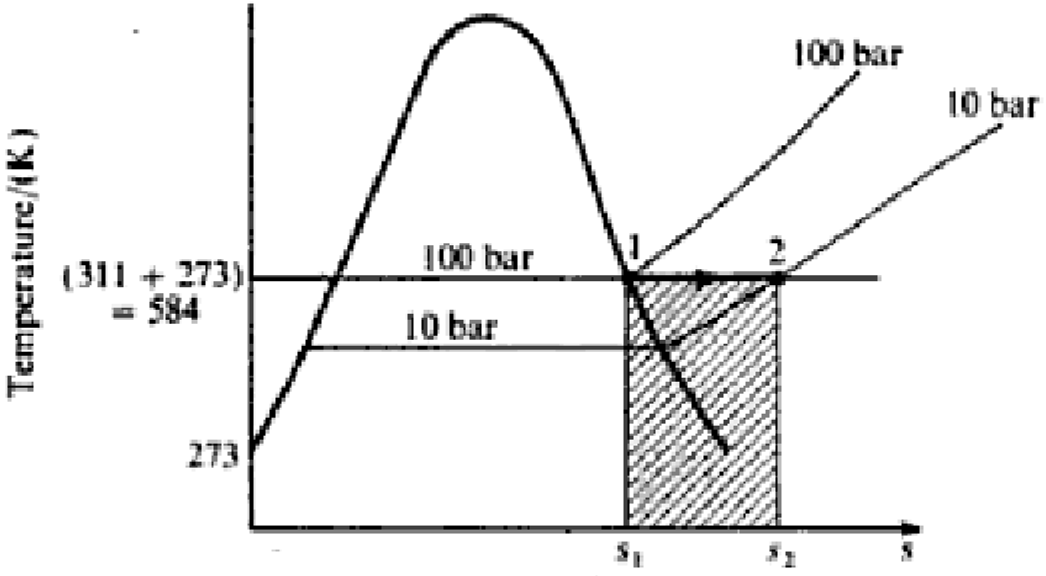
$X_1=1$ $P_1=100$ bar $T_1=T_2$ $P_2=10$ bar

$Q=?$ $W=?$

ENTROPY

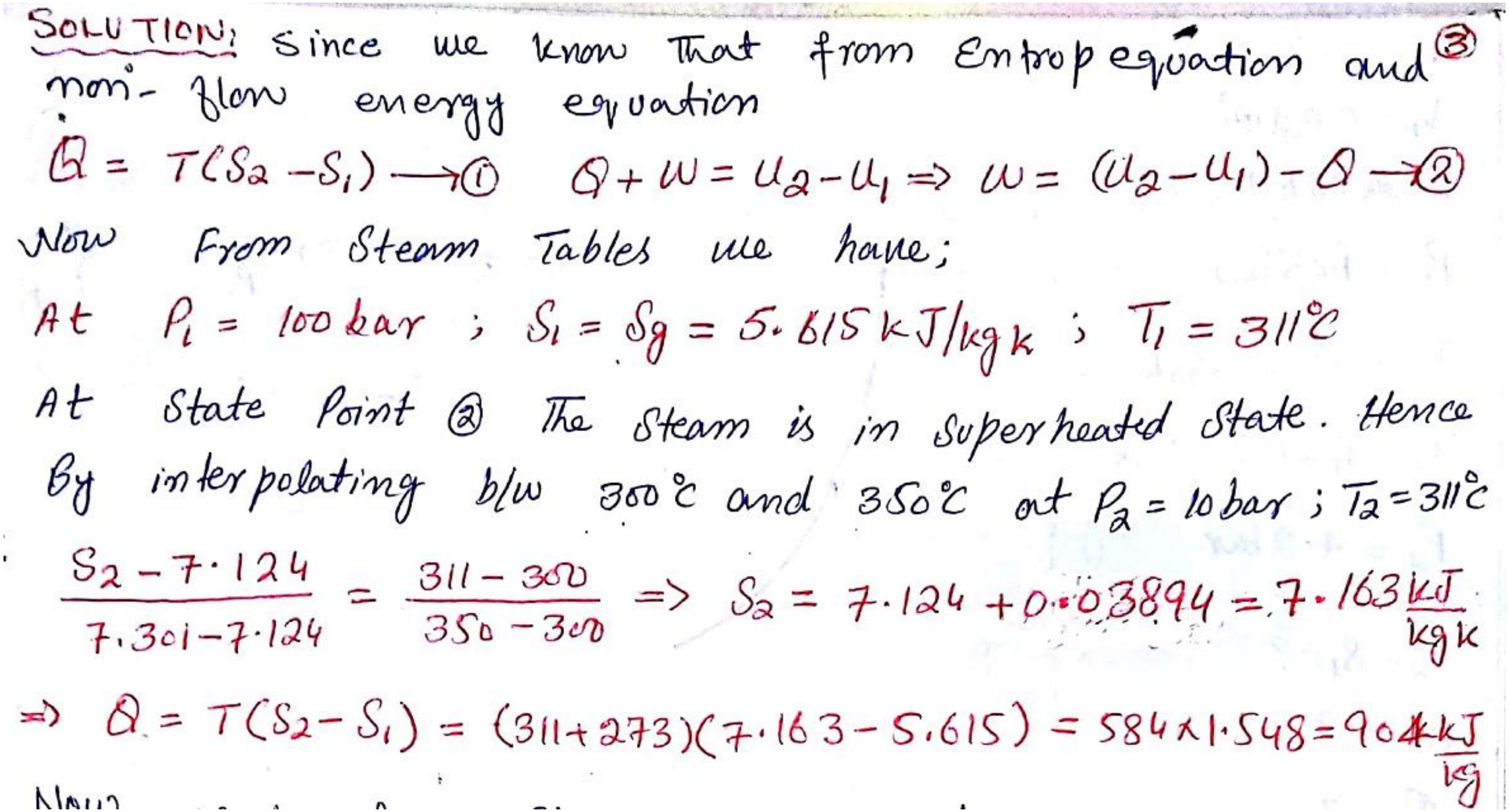
Solution:

The process is shown in Fig. 4.15, the shaded area representing the heat supplied.



ENTROPY

Solution:



SOLUTION: Since we know That from Entrop equation and ③ non-flow energy equation

$Q = T(S_2 - S_1) \longrightarrow ①$ $\quad Q + W = U_2 - U_1 \Rightarrow W = (U_2 - U_1) - Q \longrightarrow ②$

Now From Steam Tables we have;

At $P_1 = 100$ bar ; $S_1 = S_g = 5.615$ kJ/kg K ; $T_1 = 311°C$

At State Point ② The Steam is in Superheated State. Hence By interpolating b/w 300°C and 350°C at $P_2 = 10$ bar ; $T_2 = 311°C$

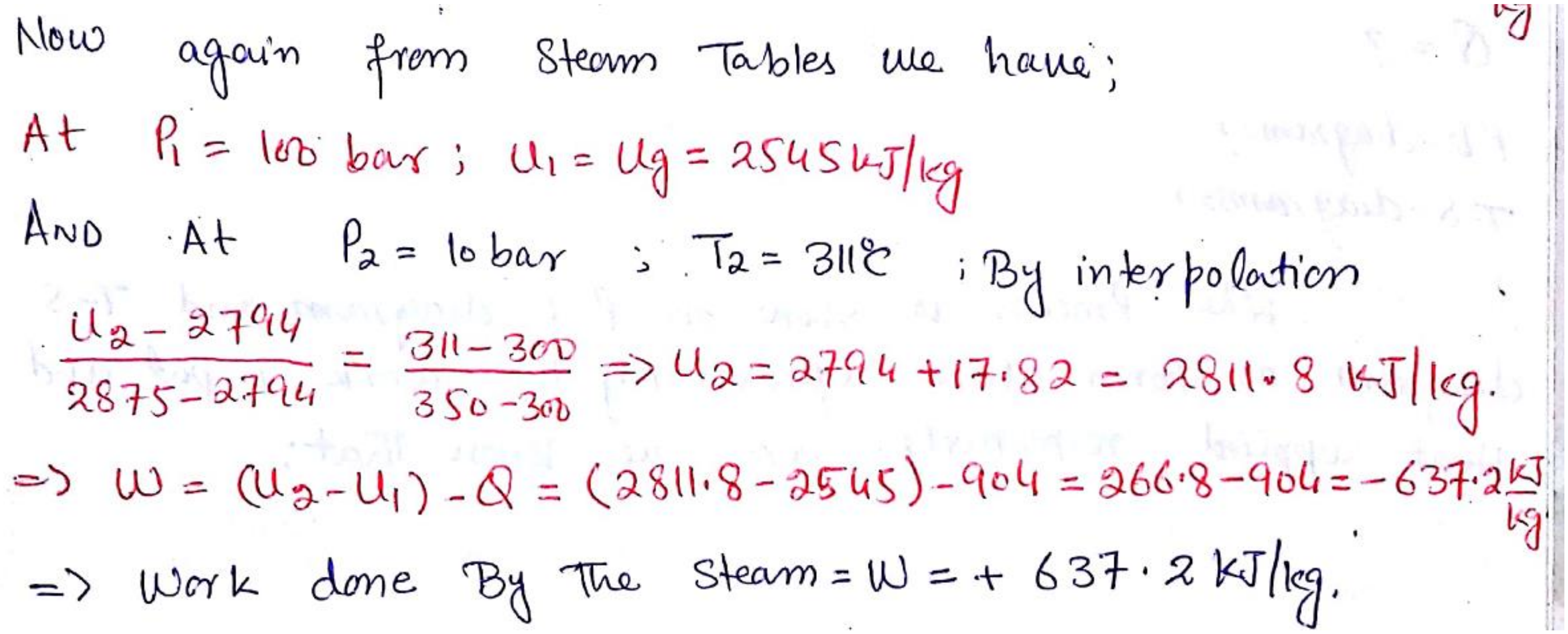
$$\frac{S_2 - 7.124}{7.301 - 7.124} = \frac{311 - 300}{350 - 300} \Rightarrow S_2 = 7.124 + 0.03894 = 7.163 \frac{kJ}{kg\,K}$$

$$\Rightarrow Q = T(S_2 - S_1) = (311 + 273)(7.163 - 5.615) = 584 \times 1.548 = 904 \frac{kJ}{kg}$$

Now

ENTROPY

Solution:



Now again from Steam Tables we have;

At $P_1 = 100$ bar ; $u_1 = u_g = 2545$ kJ/kg

And At $P_2 = 10$ bar ; $T_2 = 311°C$; By interpolation

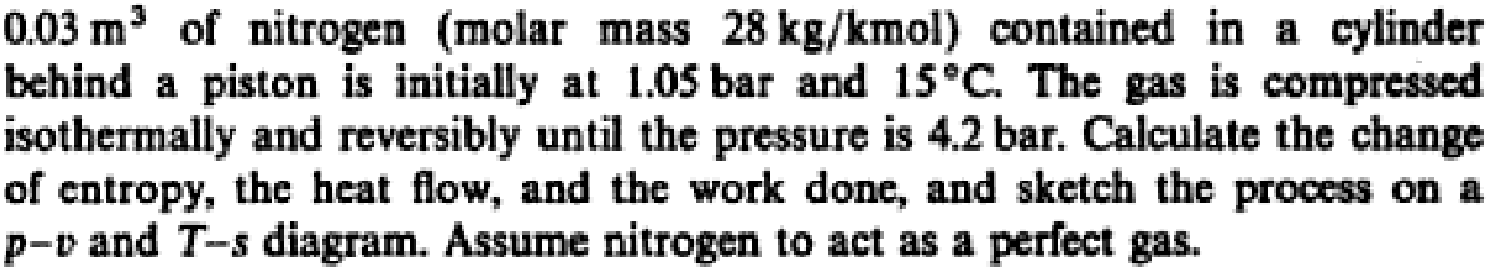
$$\frac{u_2 - 2794}{2875 - 2794} = \frac{311 - 300}{350 - 300} \Rightarrow u_2 = 2794 + 17.82 = 2811.8 \text{ kJ/kg.}$$

$$\Rightarrow W = (u_2 - u_1) - Q = (2811.8 - 2545) - 904 = 266.8 - 904 = -637.2 \frac{\text{kJ}}{\text{kg}}$$

$\Rightarrow$ Work done By The Steam $= W = +637.2$ kJ/kg.

ENTROPY

Example 4.5:



0.03 m^3 of nitrogen (molar mass 28 kg/kmol) contained in a cylinder behind a piston is initially at 1.05 bar and 15 °C. The gas is compressed isothermally and reversibly until the pressure is 4.2 bar. Calculate the change of entropy, the heat flow, and the work done, and sketch the process on a *p–v* and *T–s* diagram. Assume nitrogen to act as a perfect gas.

Given Data:

$V_1 = 0.03 m^3$

$\tilde{m} = 28$ kg/kmol

$P_1 = 1.05$ bar

$T_1 = 15°C = 288 k$

$T_2 = T_1 = 15°C = 288 k$

$P_2 = 4.2$ bar

$S_2 - S_1 = ?$

$W = ?$

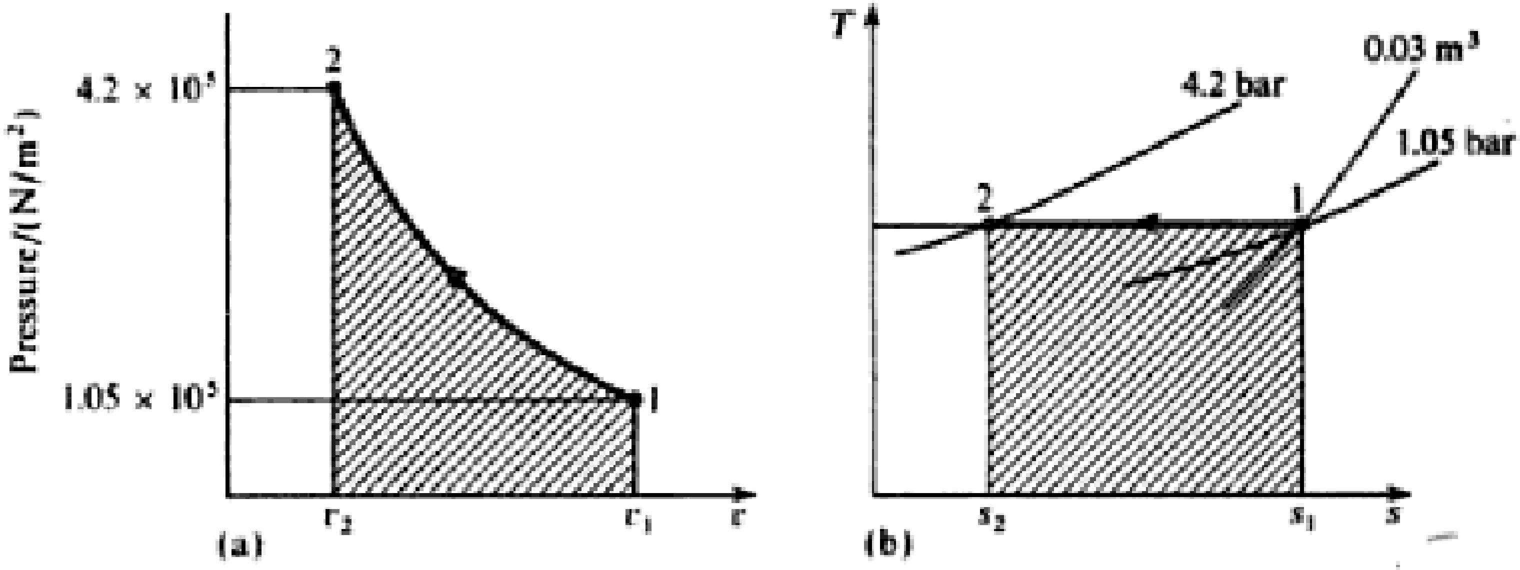
$Q = ?$

Pv-diagram = ?

T-S-diagram = ?

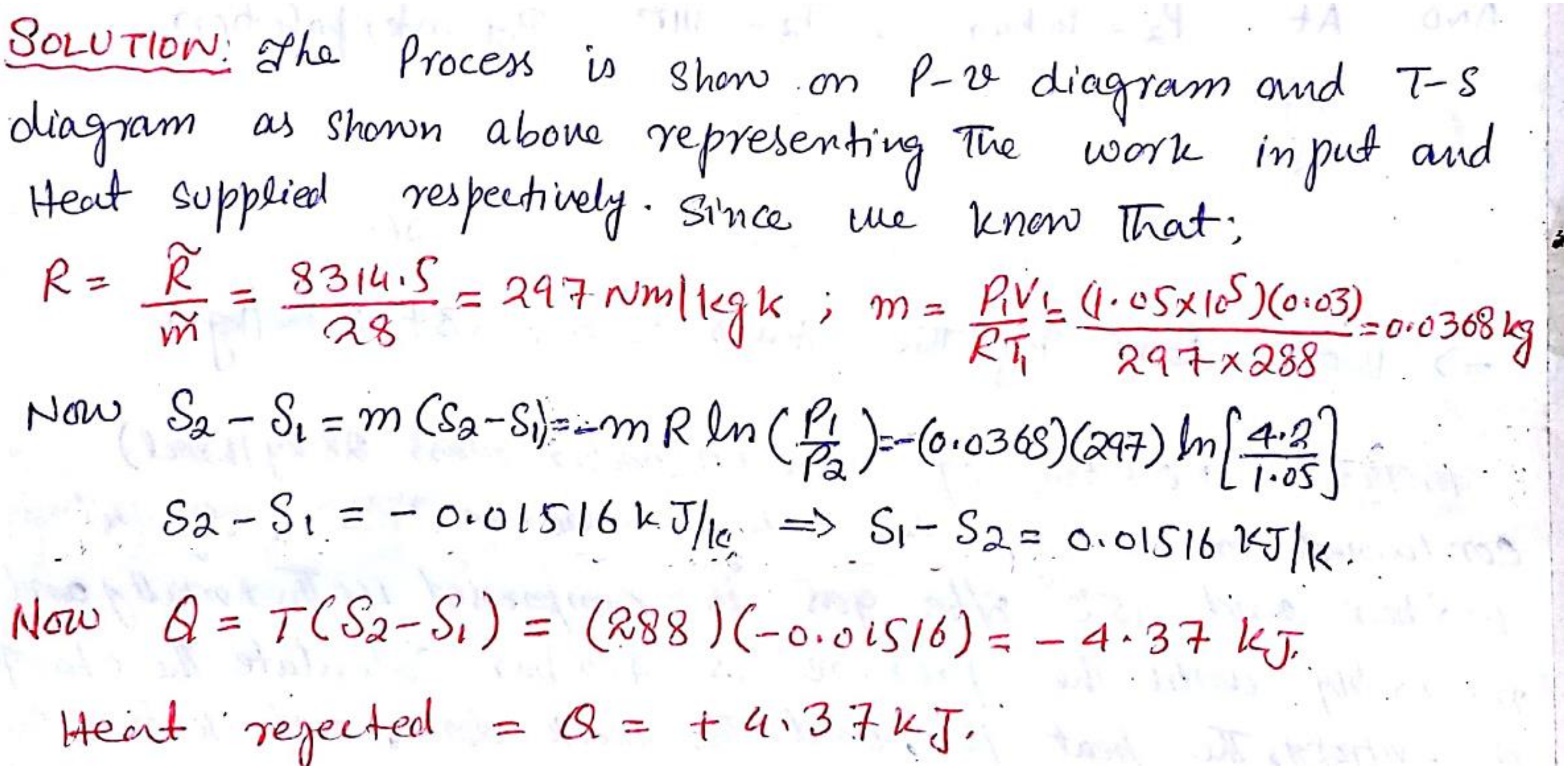
ENTROPY

Solution:



ENTROPY

Solution:



SOLUTION: The Process is show on P-v diagram and T-S diagram as shown above representing the work input and Heat supplied respectively. Since we know that;

$$R = \frac{\tilde{R}}{\tilde{m}} = \frac{8314.5}{28} = 297 \text{ Nm/kgK} \; ; \; m = \frac{P_1V_1}{RT_1} = \frac{(1.05\times10^5)(0.03)}{297\times288} = 0.0368 \text{ kg}$$

Now $S_2 - S_1 = m(s_2 - s_1) = -mR\ln\left(\frac{P_1}{P_2}\right) = -(0.0368)(297)\ln\left[\frac{4.2}{1.05}\right]$

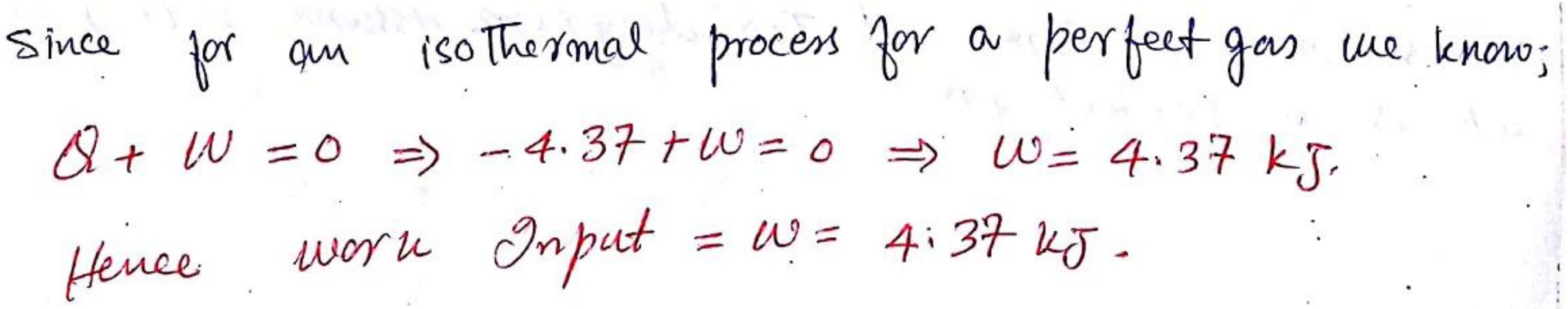
$S_2 - S_1 = -0.01516 \text{ kJ/K} \Rightarrow S_1 - S_2 = 0.01516 \text{ kJ/K}.$

Now $Q = T(S_2 - S_1) = (288)(-0.01516) = -4.37 \text{ kJ}.$

Heat rejected $= Q = +4.37 \text{ kJ}.$

ENTROPY

Solution:



Since for an isothermal process for a perfect gas we know;

$Q + W = 0 \Rightarrow -4.37 + W = 0 \Rightarrow W = 4.37 \text{ kJ}.$

Hence work Input $= W = 4.37$ kJ.

ENTROPY

Example 4.6:

Steam at 100 bar, 375 °C expands isentropically in a cylinder behind a piston to a pressure of 10 bar. Calculate the work done per kilogram of steam.

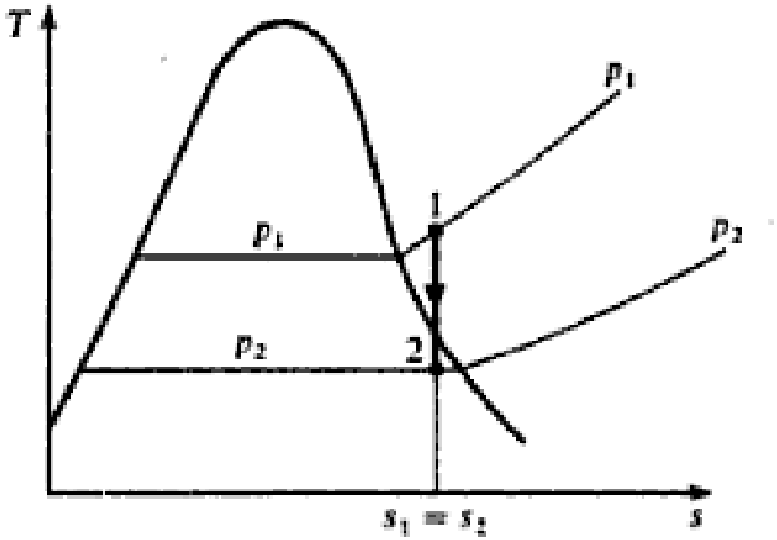
Given Data:

$P_1 = 100 \text{bar}$

$T_1 = 375^{\circ}C$

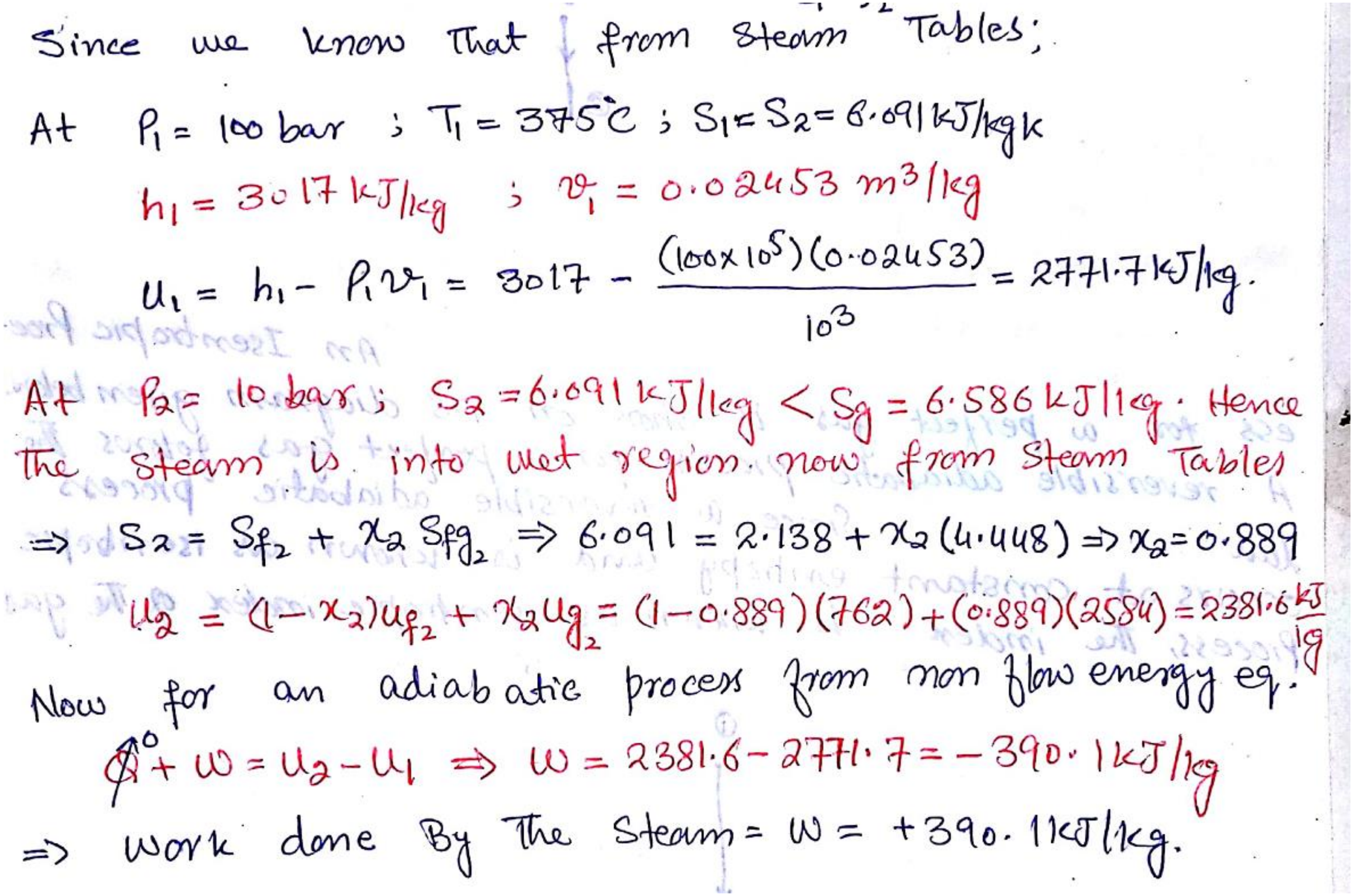
$P_2 = 10 \text{bar}$

$W = ?$



ENTROPY

Solution:



Since we know That from Steam Tables;

At $P_1 = 100$ bar ; $T_1 = 375°C$; $S_1 = S_2 = 6.091$ kJ/kgK

$h_1 = 3017$ kJ/kg ; $v_1 = 0.02453$ m³/kg

$$u_1 = h_1 - P_1 v_1 = 3017 - \frac{(100\times10^5)(0.02453)}{10^3} = 2771.7 \text{ kJ/kg}.$$

At $P_2 = 10$ bar ; $S_2 = 6.091$ kJ/kg $< S_g = 6.586$ kJ/kg. Hence The steam is into wet region. now from Steam Tables

$\Rightarrow S_2 = S_{f_2} + x_2 S_{fg_2} \Rightarrow 6.091 = 2.138 + x_2 (4.448) \Rightarrow x_2 = 0.889$

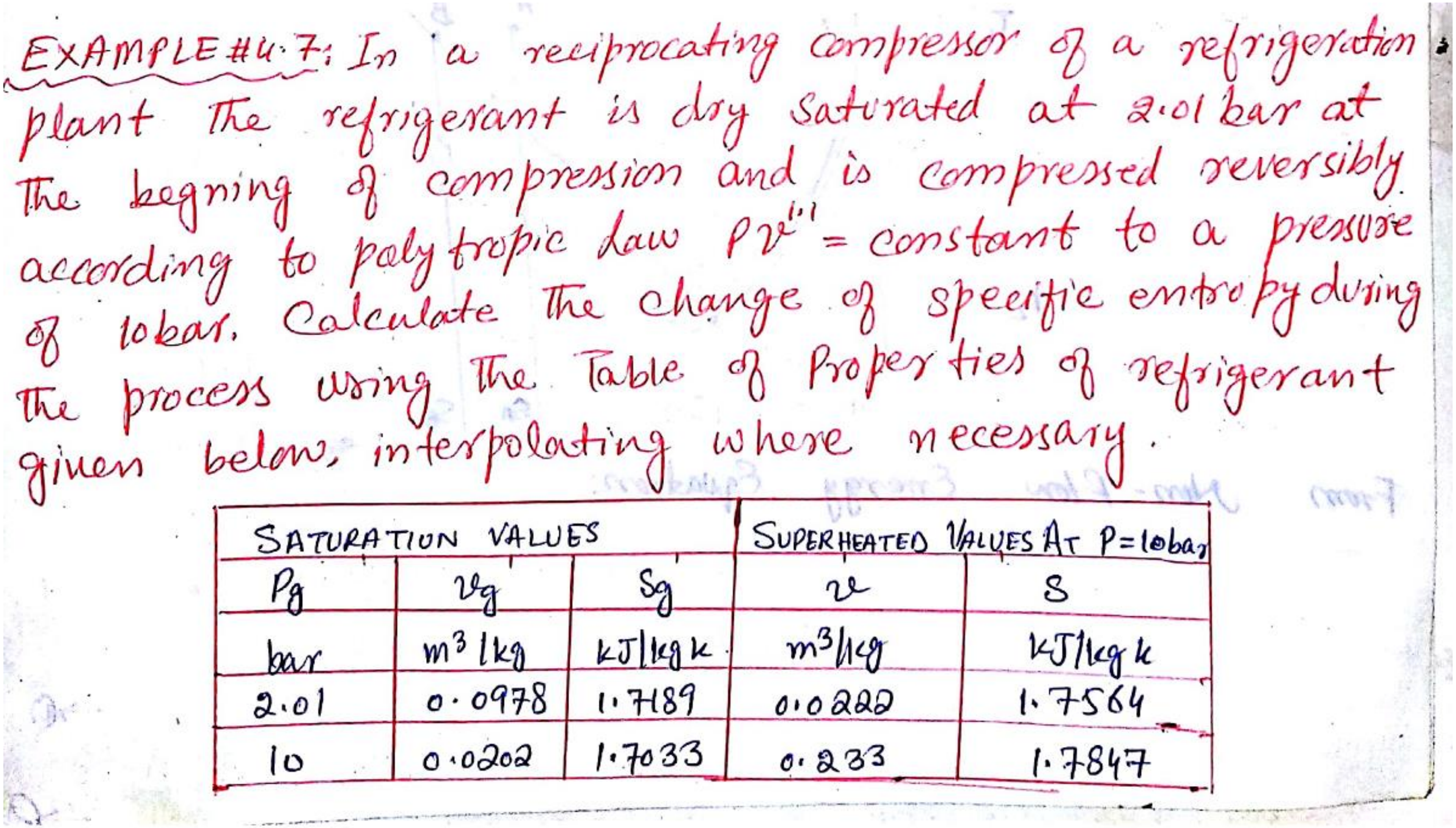
$u_2 = (1-x_2)u_{f_2} + x_2 u_{g_2} = (1-0.889)(762) + (0.889)(2584) = 2381.6 \frac{kJ}{kg}$

Now for an adiabatic process from non flow energy eq.

$\cancel{Q}^0 + W = U_2 - U_1 \Rightarrow W = 2381.6 - 2771.7 = -390.1$ kJ/kg

$\Rightarrow$ work done By The Steam $= W = +390.1$ kJ/kg.

ENTROPY



EXAMPLE#4.7: In a reciprocating compressor of a refrigeration plant the refrigerant is dry saturated at 2.01 bar at the begning of compression and is compressed reversibly according to polytropic law $pv^{1.1}$ = constant to a pressure of 10 bar. Calculate the change of specific entropy during the process using the Table of Properties of refrigerant given below, interpolating where necessary.

SATURATION VALUES			SUPERHEATED VALUES AT P=10bar	
P_g	v_g	S_g	v	S
bar	m^3/kg	kJ/kg k	m^3/kg	kJ/kg k
2.01	0.0978	1.7189	0.0222	1.7564
10	0.0202	1.7033	0.233	1.7847

ENTROPY

GIVEN DATA:

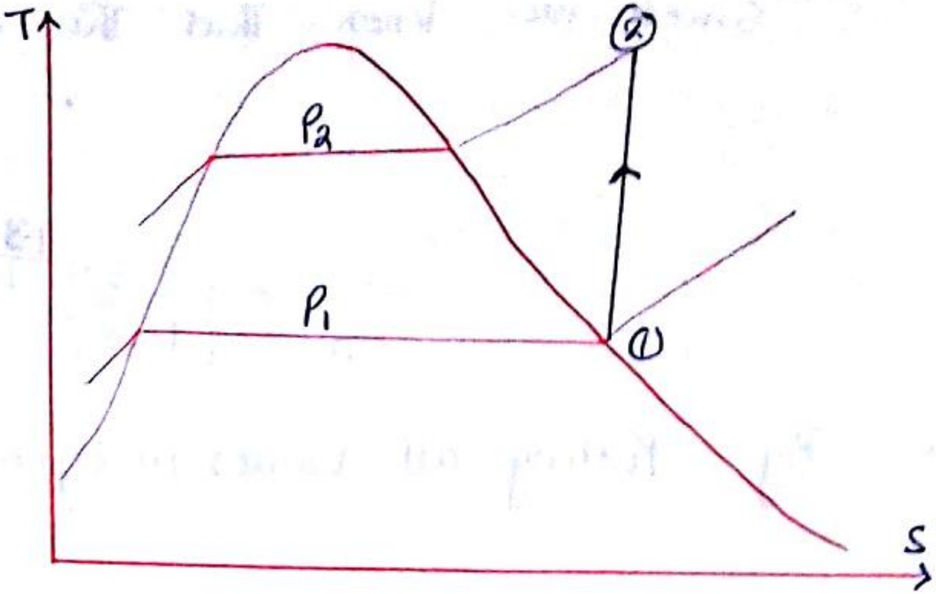
$P_1 = 2.01$ bar

$x_1 = 1$

$P_2 = 10$ bar

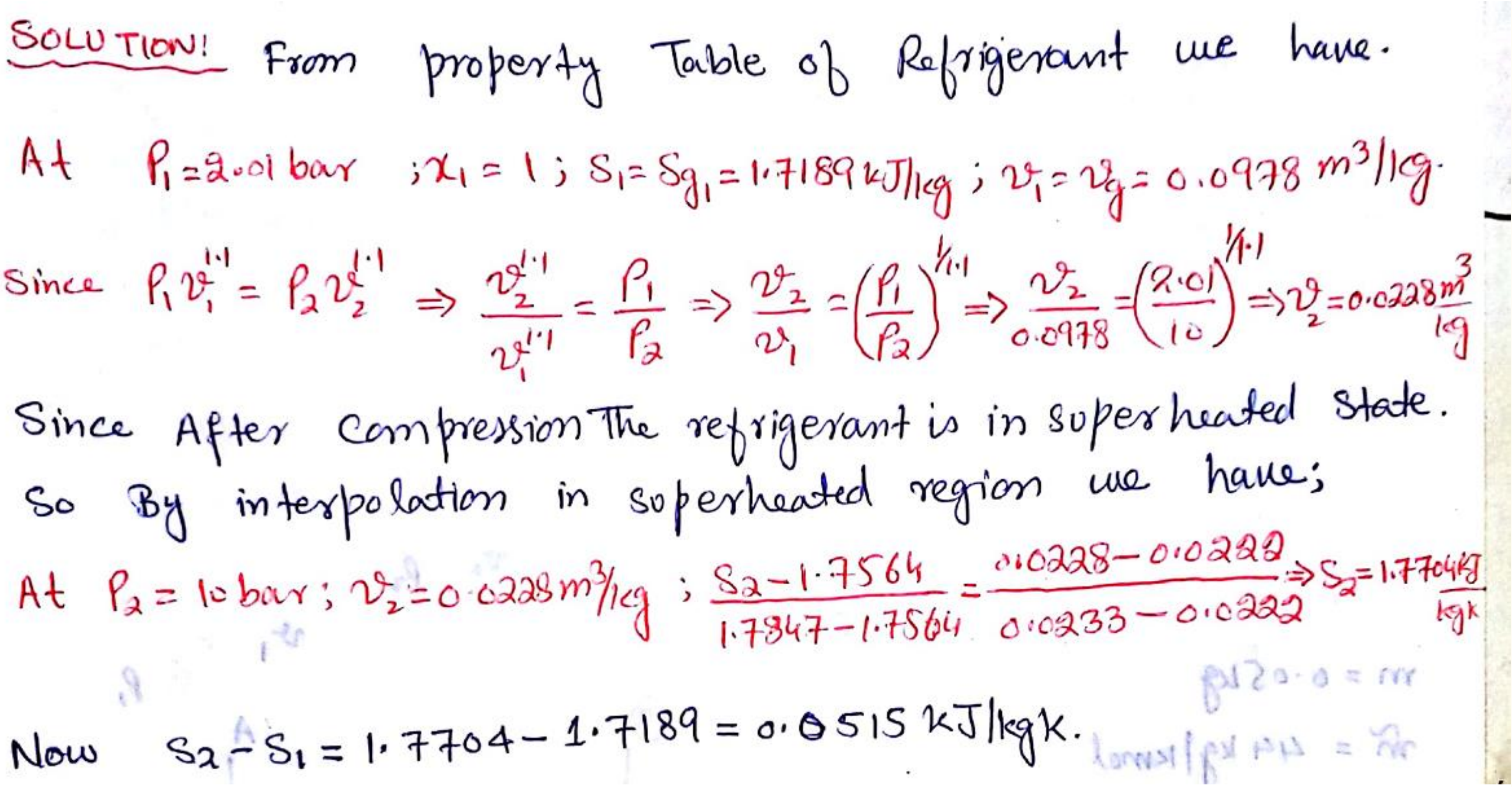
$pv^{1.1} = \text{constant}.$

$S_2 - S_1 = ?$



ENTROPY

Solution:



SOLUTION: From property Table of Refrigerant we have.

At $P_1 = 2.01$ bar ; $x_1 = 1$; $S_1 = S_{g_1} = 1.7189$ kJ/kg ; $v_1 = v_g = 0.0978$ m³/kg.

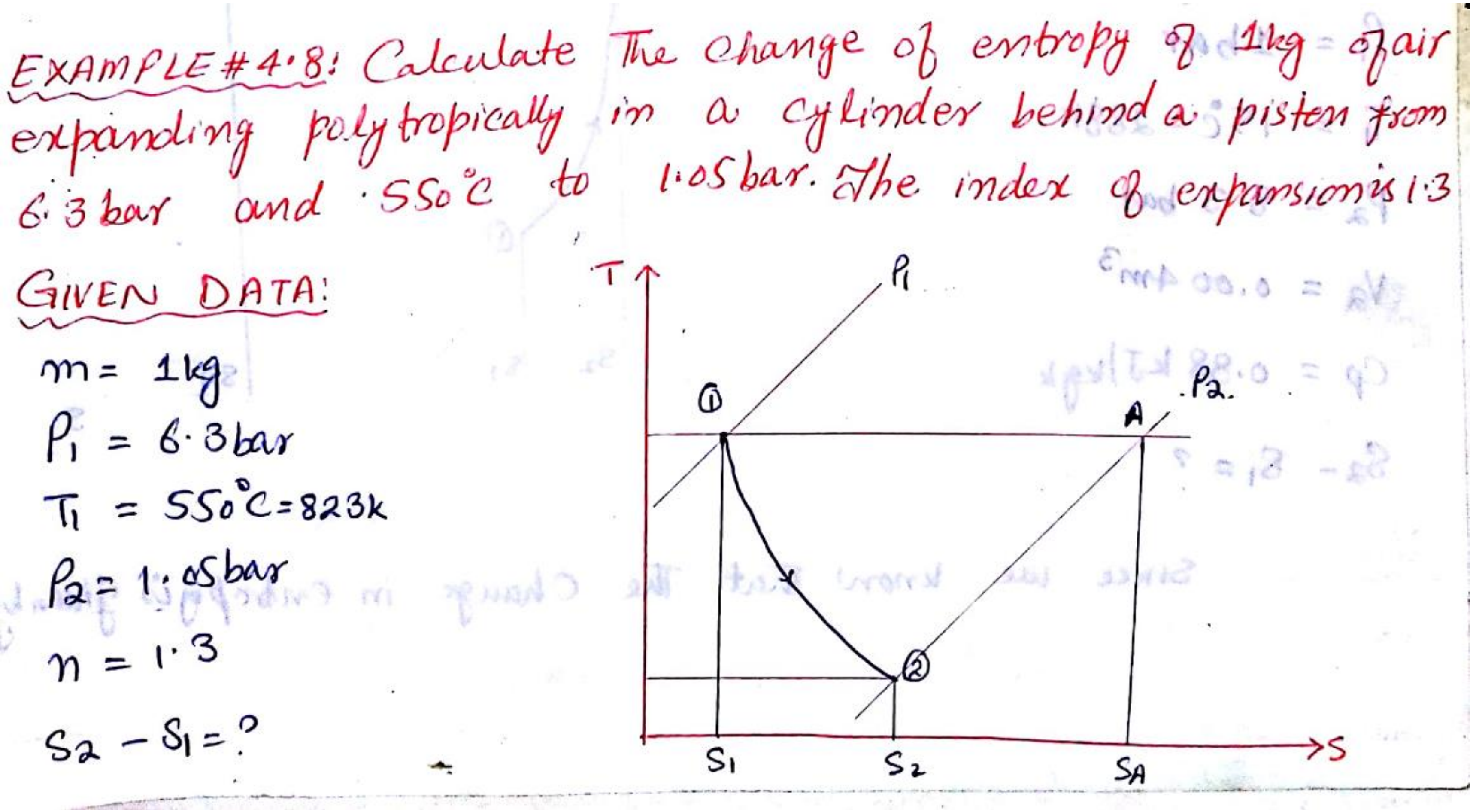
Since $P_1 v_1^{1.1} = P_2 v_2^{1.1} \Rightarrow \frac{v_2^{1.1}}{v_1^{1.1}} = \frac{P_1}{P_2} \Rightarrow \frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{1/1.1} \Rightarrow \frac{v_2}{0.0978} = \left(\frac{2.01}{10}\right)^{1/1.1} \Rightarrow v_2 = 0.0228 \frac{m^3}{kg}$

Since After compression The refrigerant is in superheated State.
So By interpolation in superheated region we have;

At $P_2 = 10$ bar ; $v_2 = 0.0228$ m³/kg ; $\frac{S_2 - 1.7564}{1.7847 - 1.7564} = \frac{0.0228 - 0.0222}{0.0233 - 0.0222} \Rightarrow S_2 = 1.7704 \frac{kJ}{kgK}$

Now $S_2 - S_1 = 1.7704 - 1.7189 = 0.0515$ kJ/kgK.

ENTROPY



EXAMPLE#4.8: Calculate The change of entropy of 1kg of air expanding polytropically in a cylinder behind a piston from 6.3 bar and 550°c to 1.05 bar. The index of expansion is 1.3

GIVEN DATA:

$m = 1\,kg$

$P_1 = 6.3\,bar$

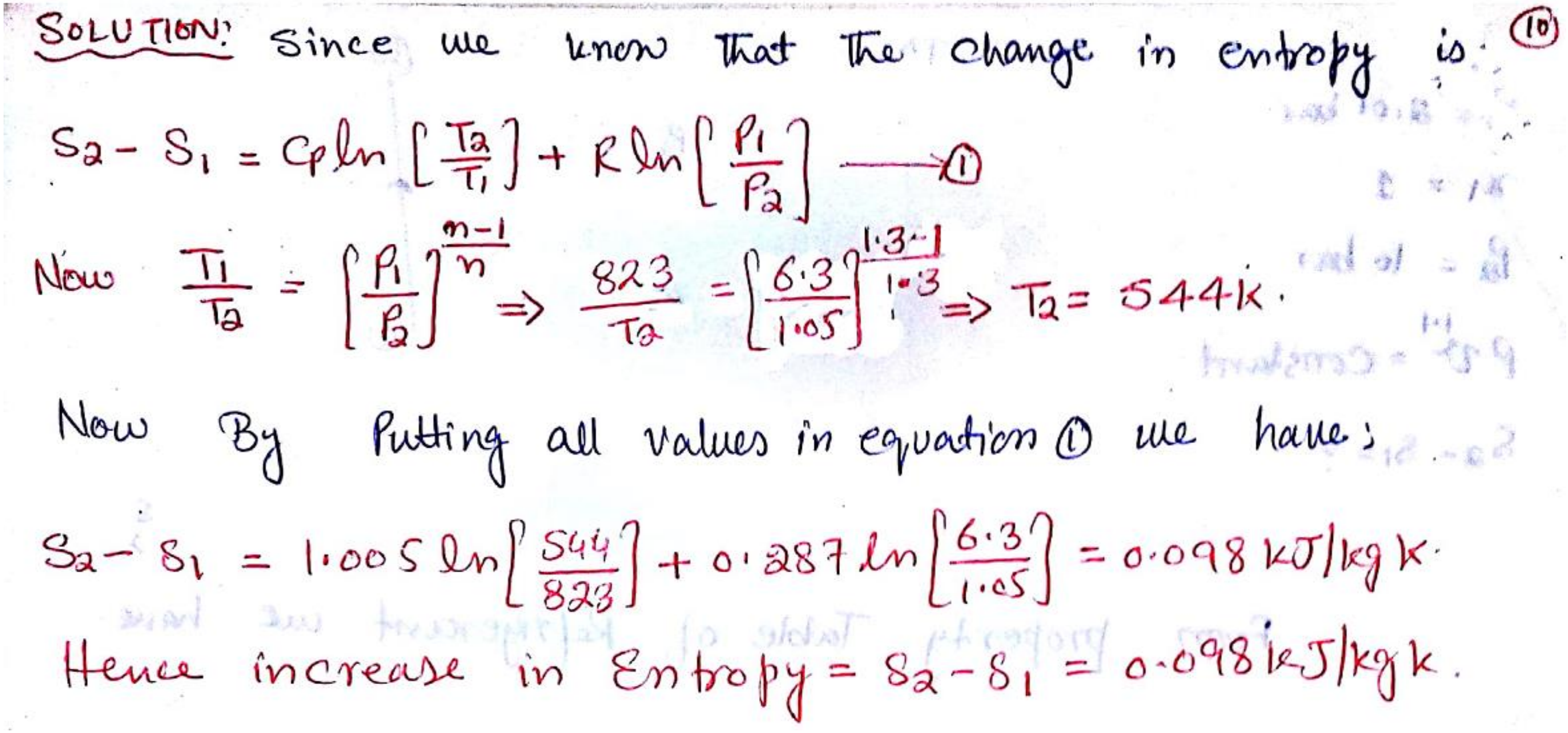
$T_1 = 550^\circ C = 823\,K$

$P_2 = 1.05\,bar$

$n = 1.3$

$S_2 - S_1 = ?$

ENTROPY



SOLUTION: Since we know that the change in entropy is: ⑩

$$S_2 - S_1 = C_p \ln\left[\frac{T_2}{T_1}\right] + R\ln\left[\frac{P_1}{P_2}\right] \longrightarrow ①$$

Now $\frac{T_1}{T_2} = \left[\frac{P_1}{P_2}\right]^{\frac{n-1}{n}} \Rightarrow \frac{823}{T_2} = \left[\frac{6.3}{1.05}\right]^{\frac{1.3-1}{1.3}} \Rightarrow T_2 = 544K.$

Now By Putting all values in equation ① we have;

$$S_2 - S_1 = 1.005 \ln\left[\frac{544}{823}\right] + 0.287 \ln\left[\frac{6.3}{1.05}\right] = 0.098 \text{ kJ/kg K}.$$

Hence increase in Entropy $= S_2 - S_1 = 0.098$ kJ/kgK.

ENTROPY

EXAMPLE#4.9: 0.05 kg of Carbon dioxide (molar mass 44 kg/kmol) is compressed from 1 bar, 15°C, until The pressure is 8.3 bar and The volume is Then $0.004 m^3$. Calculate The change of Entropy. Take C_p for Carbon dioxide as 0.88 kJ/kgK, and assume Carbon dioxide to be a perfect gas.

GIVEN DATA:

$m = 0.05 kg$

$\tilde{m} = 44 kg/kmol$

$P_1 = 1 bar$

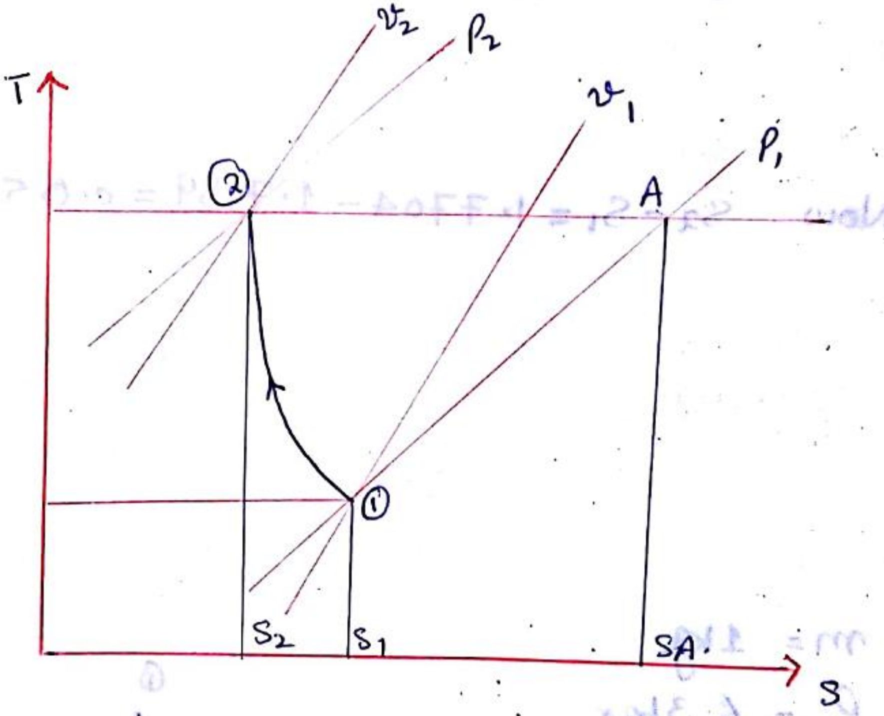
$T_1 = 15°C = 288K$

$P_2 = 8.3 bar$

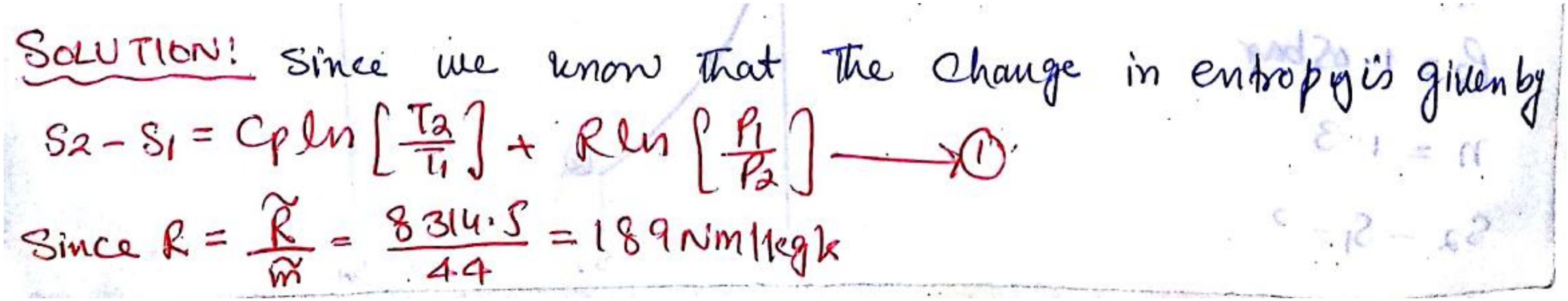
$V_2 = 0.004 m^3$

$C_p = 0.88 kJ/kgk$

$S_2 - S_1 = ?$



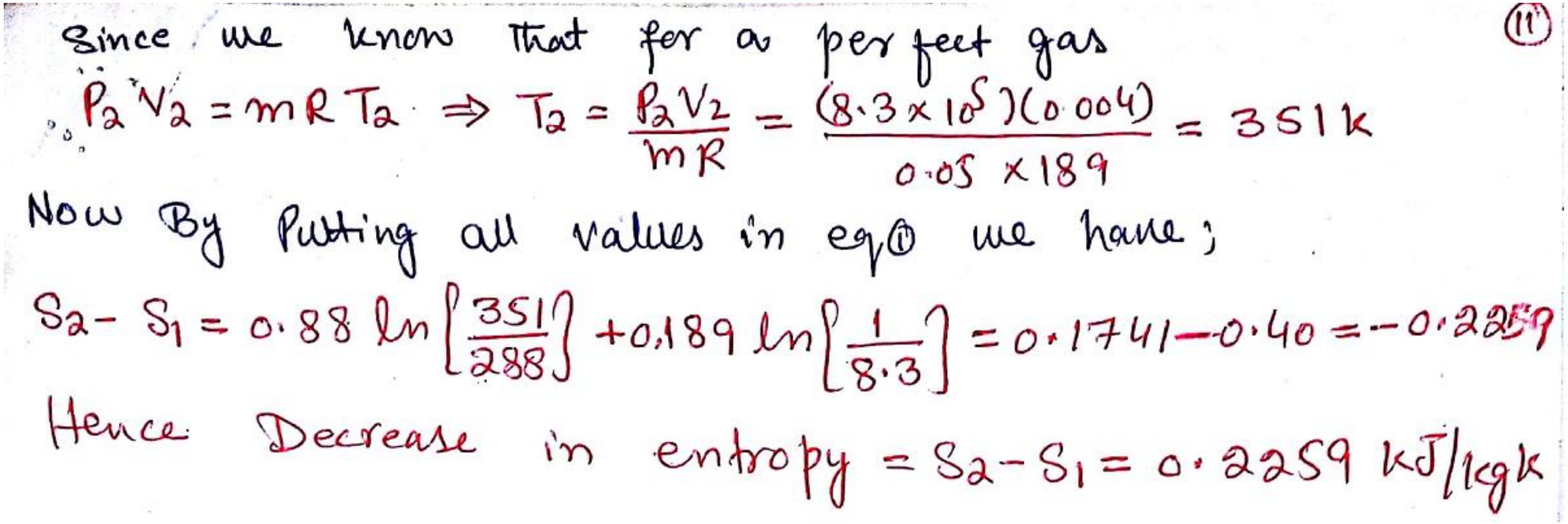
ENTROPY



SOLUTION: Since we know that the change in entropy is given by

$$S_2 - S_1 = C_p \ln\left[\frac{T_2}{T_1}\right] + R\ln\left[\frac{P_1}{P_2}\right] \longrightarrow ①$$

Since $R = \frac{\tilde{R}}{\tilde{m}} = \frac{8314.5}{4.4} = 189\,Nm/kgk$



(II)

Since we know that for a perfect gas

$$P_2V_2 = mRT_2 \Rightarrow T_2 = \frac{P_2V_2}{mR} = \frac{(8.3\times10^5)(0.004)}{0.05\times189} = 351k$$

Now By Putting all values in eq① we have;

$$S_2 - S_1 = 0.88\ln\left[\frac{351}{288}\right] + 0.189\ln\left[\frac{1}{8.3}\right] = 0.1741 - 0.40 = -0.2259$$

Hence Decrease in entropy $= S_2 - S_1 = 0.2259\ kJ/kgk$

ENTROPY

Exercise Problems: 4.1, 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8, 4.9, 4.10, 4.11, 4.12, 4.13, 4.14, 4.15, 4.16, 4.17, 4.18


Thank You!