

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ



# **Applied Thermodynamics**

**Week\_12**

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## *ENTROPY*

- Entropy is a thermodynamic property that is a measure of the energy not available for useful work in a thermodynamic process, such as in energy conversion devices, engines, or machines.
- Such devices can only be driven by convertible energy and have a theoretical maximum efficiency when converting energy to work.
- During this work entropy accumulates in the system but should be removed by dissipation in the form of waste heat.
- The concept of entropy is defined by the second law of thermodynamics, which states that the entropy of a closed system always increases.

## **ENTROPY**

- Thus, entropy is also measure of the tendency of a process, such as a chemical reaction, to be *entropically favored*, or to proceed in a particular direction.
- It determines that thermal energy always flows spontaneously from regions of higher temperature to regions of lower temperature, in the form of heat.
- These processes reduce the state of order of the initial systems, and therefore **entropy is an expression of disorder or randomness**.
- Thermodynamic entropy has the dimension of energy divided by temperature, and a unit of joules per kelvin (J/K) in the international system of units.

# ENTROPY

- Consider a reversible adiabatic process for any system on a  $p$ - $v$  diagram.
- This is represented by line AB on Fig. 4.4
- Let us suppose that it is possible for the system to undergo a reversible isothermal process at temperature  $T_1$  from B to C and then be restored to its original state by a second reversible adiabatic process from C to A.

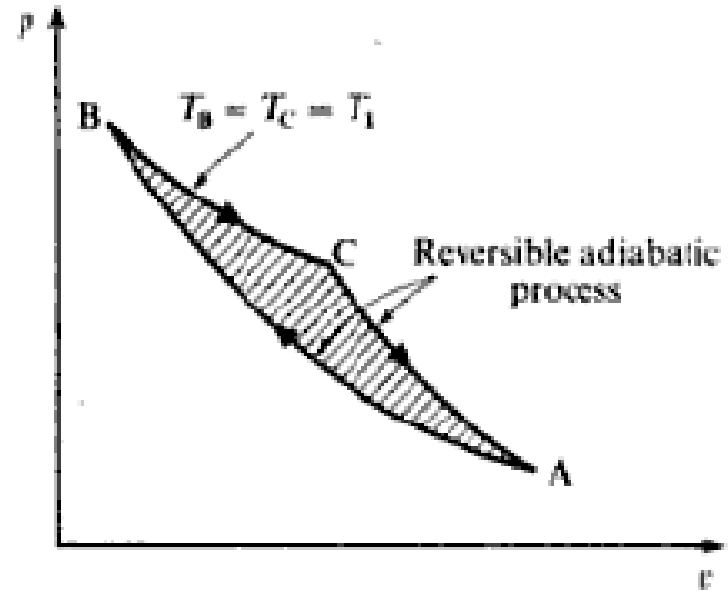


Fig.4.4: Hypothetical cycle on a  $p$ -  $v$  diagram

# ENTROPY

- Now by definition, an adiabatic process is one in which no heat flows to or from the system.
- Hence, the only heat transferred is from B to C during the isothermal process. The work done by the system is given by the enclosed area.
- We therefore have a system undergoing a cycle and developing a net work output while drawing heat from a reservoir at one fixed temperature.

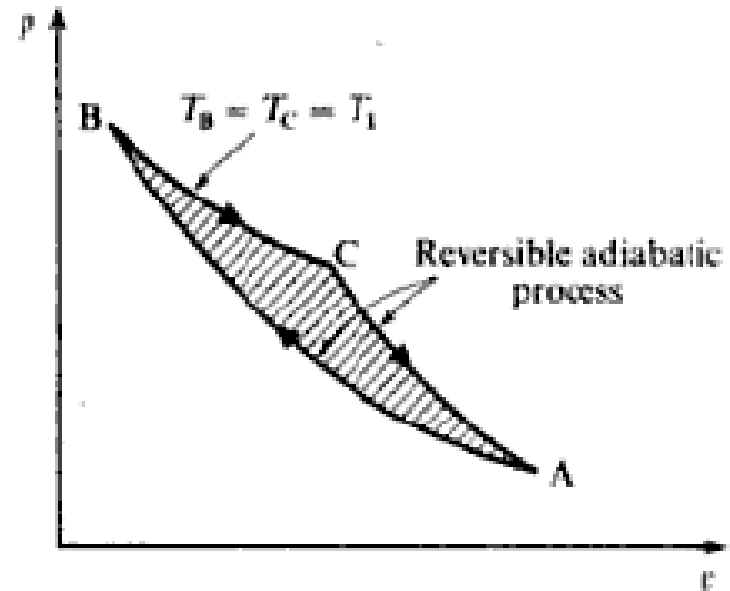


Fig.4.4: Hypothetical cycle on a  $p$ - $v$  diagram

# ENTROPY

- This is impossible because it violates the second law.
- Therefore, the original supposition is wrong, and it is not possible to have two reversible adiabatic processes passing through the same state A.

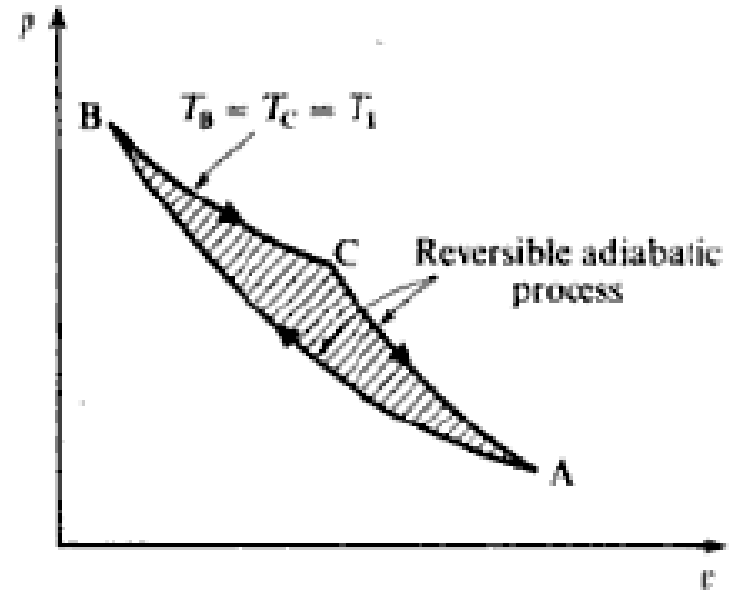


Fig.4.4: Hypothetical cycle on a  $p$ -  $v$  diagram

## ENTROPY

- Now one of the characteristics of a property of a system is that there is one unique line which represents a value of the property on a diagram of properties. (For example, the line BC on Fig. represents the isothermal process at  $T_1$ .)
- Hence there must be a property represented by a reversible adiabatic process. This property is called entropy,  $s$ .

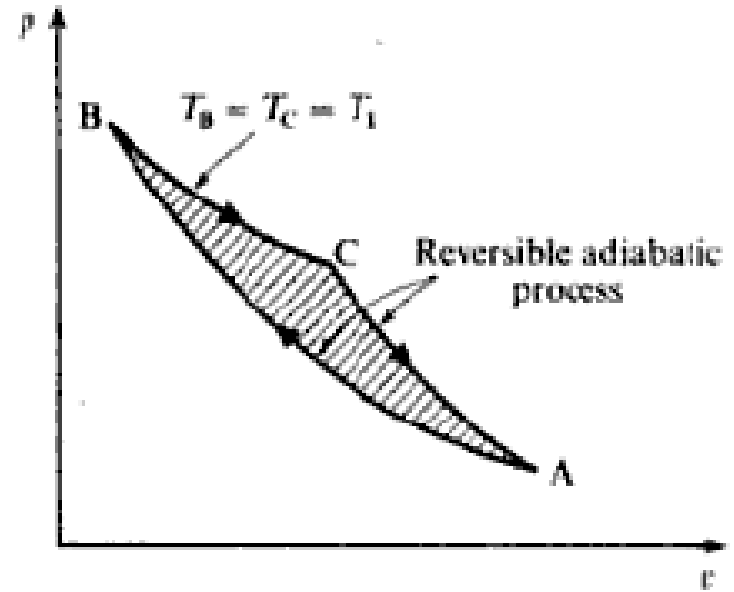
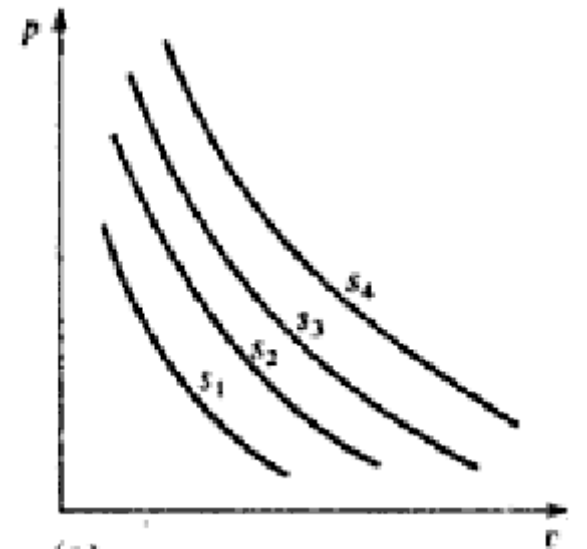


Fig.4.4: Hypothetical cycle on a  $p$ - $v$  diagram

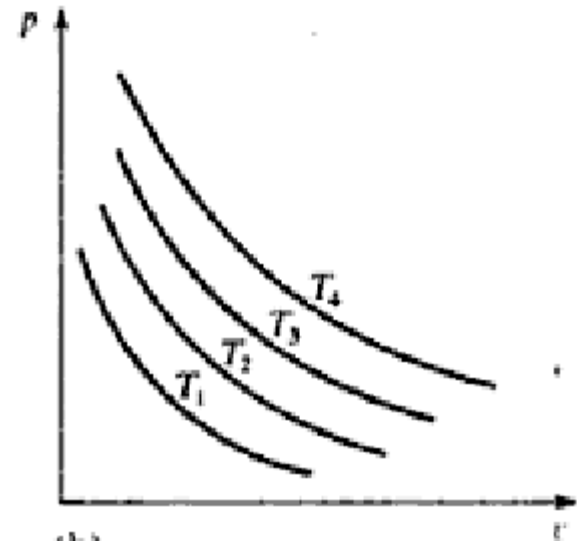


## ENTROPY

- It follows that there is no change of entropy in a reversible adiabatic process. Each reversible adiabatic process represents a unique value of entropy.
- On a  $p-v$  diagram a series of reversible adiabatic processes appear as shown in Fig. 4.5(a). each line representing one value of entropy. This is similar to Fig. 4.5(b) in which a series of isothermals is drawn, each representing one " value of temperature.



(a)



(b)

Fig.5: A series of constant entropy and constant temperature lines on a  $p-v$  diagram

## ENTROPY

Starting with the non-flow energy equation for a reversible process;

$$dQ = du + pdv$$

And for a perfect gas;

$$dQ = c_v dT + RT \frac{dv}{v}$$

This equation can be integrated after dividing through by T1.

$$\frac{dQ}{T} = \frac{c_v dT}{T} + \frac{R dv}{v}$$

Also, for an adiabatic process,  $dQ=0$

$$\frac{dQ}{T} = \frac{c_v dT}{T} + \frac{R dv}{c} = 0 \rightarrow (4.6)$$

We can say, therefore,  $dQ/T = 0$  for a reversible adiabatic process. For any other reversible process  $dQ/T \neq 0$ . This result can be shown to apply to all working substances,

i.e.  $ds = \frac{dQ}{T} \rightarrow (4.7)$  for all working substances

where s is entropy.

## *ENTROPY*

- Note that since equation (4.6) is for a reversible process, then  $dQ$  in equation (4.7) is the heat added reversibly.
- The change of entropy is more important than its absolute value, and the zero of entropy can be chosen quite arbitrarily.
- Integrating equation (4.7) gives

$$s_2 - s_1 = \int_1^2 \frac{dQ}{T} \rightarrow (4.8)$$

- Considering unit mass of fluid, the units of entropy are given by kilojoules per kilogram divided by K. That is, the units of specific entropy,  $s$ , are kJ/kg K The symbol  $S$  will be used for the entropy of mass,  $m$ , of a fluid, i.e.

$$s = ms$$

Rewriting equation (4.7) we have

$$dQ = Tds$$

# ENTROPY

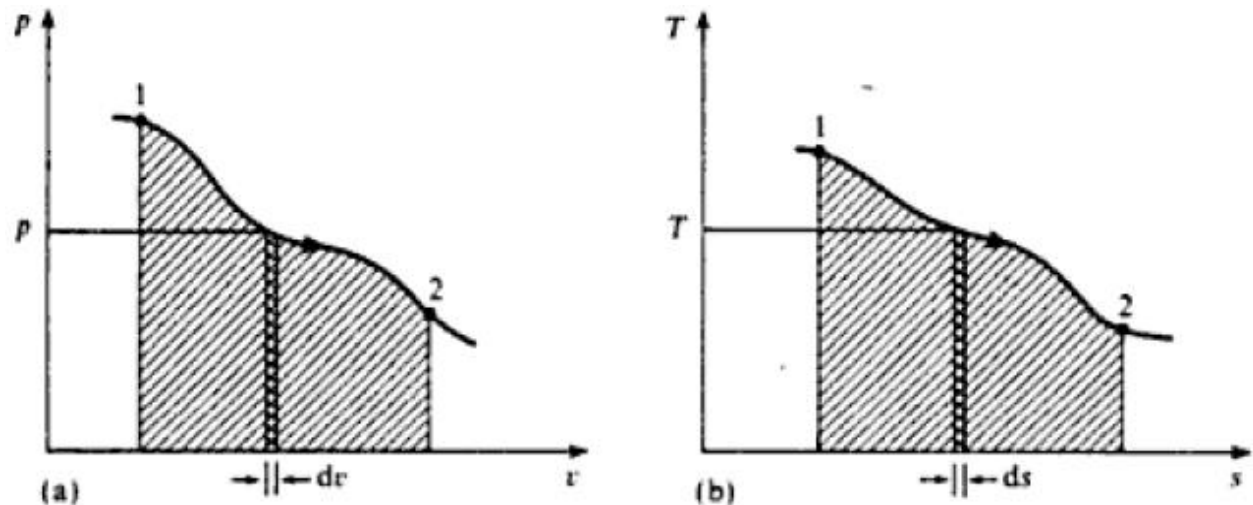
or for any reversible process

$$Q = \int_1^2 T ds \rightarrow (4.9)$$

This equation is analogous to equation,  $W = - \int_1^2 p dv$  for any reversible process

Thus, as there is a diagram on which areas represent work output in a reversible process, there is also a diagram on which areas represent heat supplied in a reversible process. These diagrams are the *p-v* and the *T-s* diagrams respectively, as shown in Figs 4.6(a) and 4.6(b).

Fig.4.6:Area under a reversible process on a p-v and on T-s diagram



# ENTROPY

## THE T-S DIAGRAM FOR VAPOUR

- The T-s diagram for steam is shown in Fig. 4.7.
- Three lines of constant pressure ( $p_1$ ,  $p_2$  and  $p_3$ ) are shown (i.e. lines ABCD, EFGH, and JKLM).
- The pressure lines in the liquid region are practically coincident with the saturated liquid line (i.e., portions AB, EF, and JK), and the difference is usually neglected.
- The pressure remains constant with temperature when the latent heat is added, hence the pressure lines are horizontal in the wet region (i.e., portions BC, FG, and KL).
- The pressure lines curve upwards in the superheat region as shown (i.e., portions CD, GH, and LM). Thus, the temperature rises as heating continues at constant pressure.
- One constant volume line (shown as a broken line) is drawn in Fig. 4.7. Lines of constant volume are concave down in the wet region and slope up more steeply than pressure lines in the superheat region.

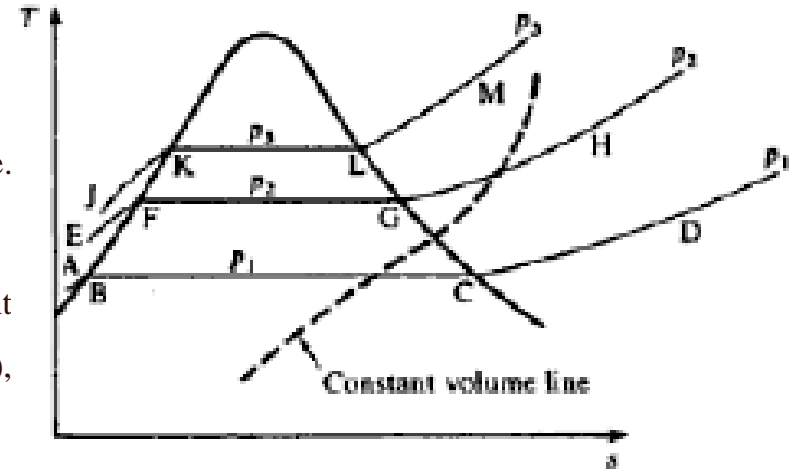


Fig. 4.7: T-s diagram for a vapour

# ENTROPY

## THE T-S DIAGRAM FOR VAPOUR

- In steam tables the entropy of the saturated liquid and the dry saturated vapor are represented by  $s_f$  and  $s_g$  respectively.
- The difference,  $s_f - s_g = s_{fg}$  is also tabulated. The entropy of wet steam is given by the entropy of the water in the mixture plus the entropy of the dry steam in the mixture.
- For wet steam with dryness fraction,  $x$ , we have;

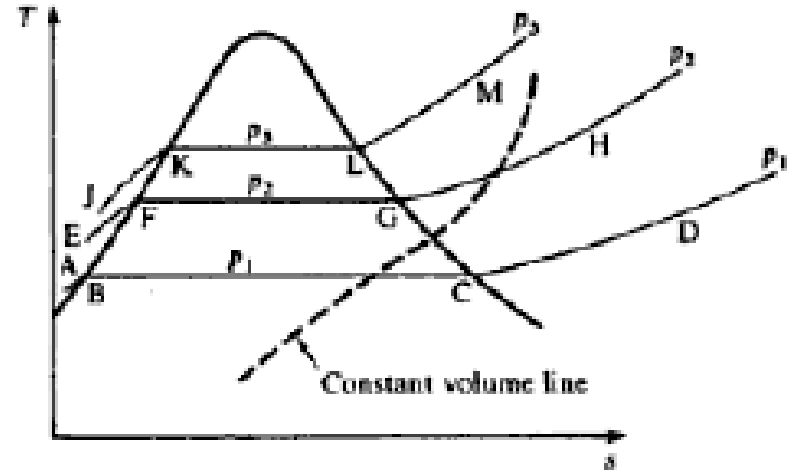


Fig. 4.7: T-s diagram for a vapour

$$s = (1 - x)s_f + xs_g \rightarrow (4.10)$$

$$\Rightarrow s = s_f + x(s_g - s_f)$$

$$\Rightarrow s = s_f + s_{fg} \rightarrow (4.11)$$

$$\Rightarrow x = \frac{s - s_f}{s_{fg}} \rightarrow (4.12)$$

# ENTROPY

## THE T-S DIAGRAM FOR VAPOUR

- It can be seen from equation (4.12) that the dryness fraction is proportional to the distance of the state point from the liquid line on a T-s diagram. For example, for state 1 on Fig. 4.8 the dryness fraction

$$x_1 = \frac{\text{Distance } F1}{\text{Distance } FG} = \frac{s - s_{f1}}{s_{fg1}}$$

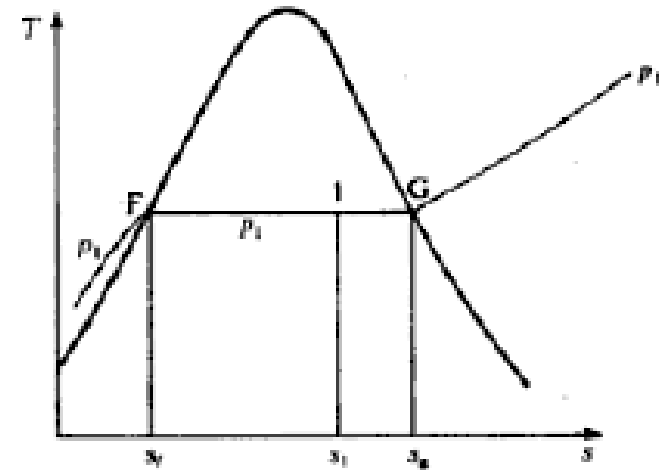


Fig. 4.8: Dryness fraction from areas on a T-s diagram

- The area under the line FG on Fig. 4.8 represents the specific enthalpy of vaporization  $h_{fg}$ . The area under line F1 is given by  $x_1 h_{fg}$ .

# ENTROPY

## THE T-S DIAGRAM FOR PERFECT GAS

- It is useful to plot lines of constant pressure and constant volume on a T-s diagram for a perfect gas.
- Since changes of entropy are of more direct application than the absolute value, the zero of entropy can be chosen at any arbitrary reference temperature and pressure.
- In Fig. 4.9 the pressure line  $p_1$  and the volume line  $v_1$  have been drawn passing through the state point 1. Note that a line of constant pressure slopes less steeply than a line of constant volume.

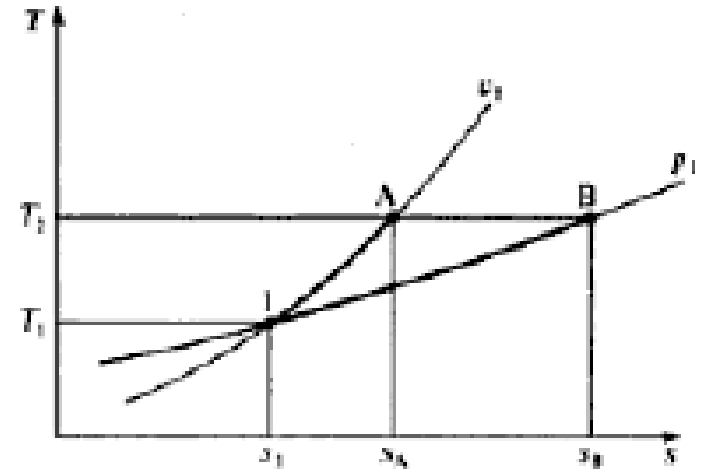


Fig. 4.9: Entropy changes at constant pressure and at constant volume for a perfect gas on a T-s diagram



# ENTROPY

## THE T-S DIAGRAM FOR PERFECT GAS

- This can be proved easily by reference to Fig. 4.9. Let points A and B be at  $T_2$  and  $v_1$  and  $T_2$  and  $p_1$  respectively as shown. Now between 1 and A from equation (4.8) we have;

$$s_A - s_1 = \int_1^A \frac{dQ}{T}$$

- Also, at a constant volume for 1 kg of gas from equation  $dQ=c_v dT$ . Therefore

$$s_A - s_1 = \int_1^A \frac{c_v dT}{T} = c_v \ln \left( \frac{T_A}{T_1} \right) = c_v \ln \left( \frac{T_2}{T_1} \right)$$

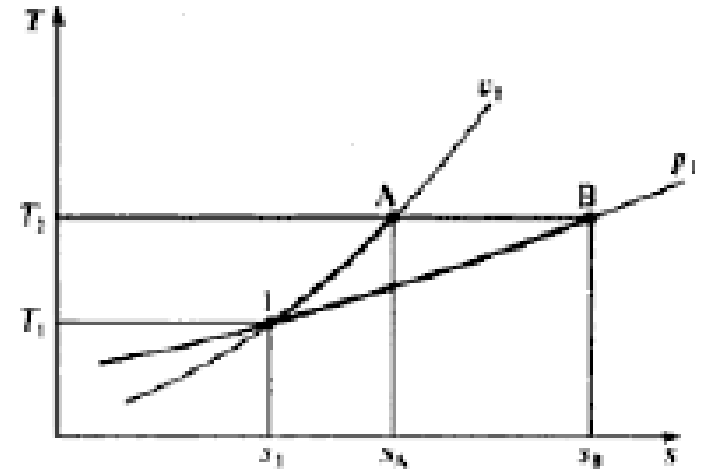


Fig. 4.9: Entropy changes at constant pressure and at constant volume for a perfect gas on a T-s diagram

# ENTROPY

## THE T-S DIAGRAM FOR PERFECT GAS

- Similarly, at constant pressure for 1 kg of gas  $dQ=c_p dT$ . Hence,

$$s_A - s_1 = \int_1^B \frac{c_p dT}{T} = c_p \ln \left( \frac{T_B}{T_1} \right) = c_p \ln \left( \frac{T_2}{T_1} \right)$$

- Now since  $c_p$  is greater than  $c_v$  for any perfect gas, then  $s_B - s_1$  is greater than  $s_A - s_1$ .
- Point A must therefore lie to the left of point B on the diagram, and hence a line of constant pressure slopes less steeply than a line of constant volume.

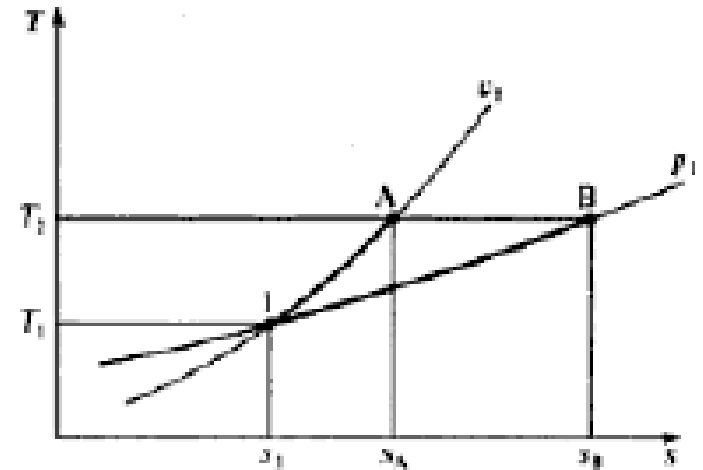


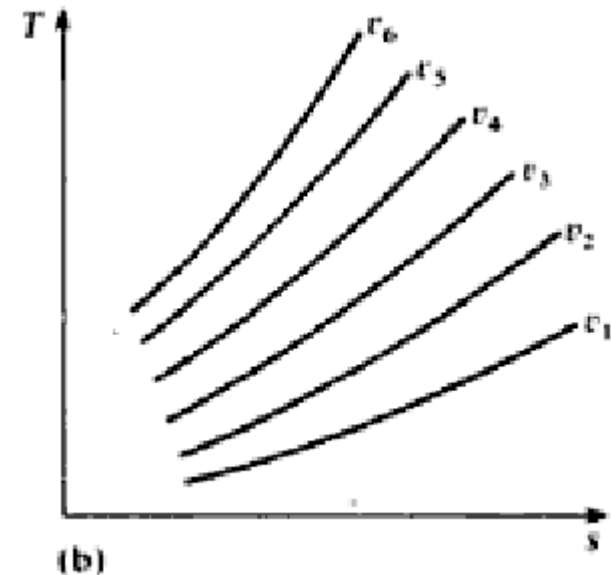
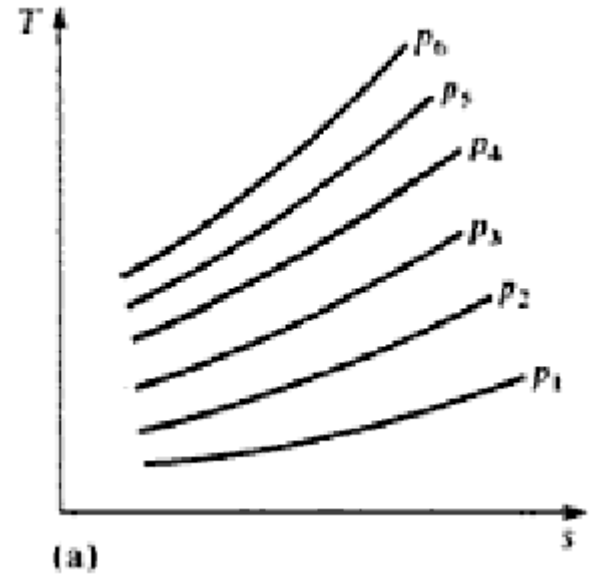
Fig. 4.9: Entropy changes at constant pressure and at constant volume for a perfect gas on a T-s diagram

# ENTROPY

## THE T-S DIAGRAM FOR PERFECT GAS

- Figure 4.12(a) shows a series of constant pressure lines on a T-s diagram, and Fig. 12(b) shows a series of constant volume lines on a T-s diagram.
- Note that in Fig. 4.12(a),  $p_6 > p_5 > p_4 > p_3$ , etc. and in Fig. 12(b),  $v_1 > v_2 > v_3$ , etc. As the pressure rises the temperature rises and the volume decreases; conversely as the pressure and temperature fall the volume increases

Fig. 4.12: Constant pressure and constant volume lines plotted on a T-s diagram for a perfect gas



## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### REVERSIBLE ISOTHERMAL PROCESS

- A reversible isothermal process will appear as a horizontal line on a T-s diagram, and the area under the line must represent the heat flow during the process.
- For example, Fig. 4.14 shows a reversible isothermal expansion of wet steam into the superheat region.

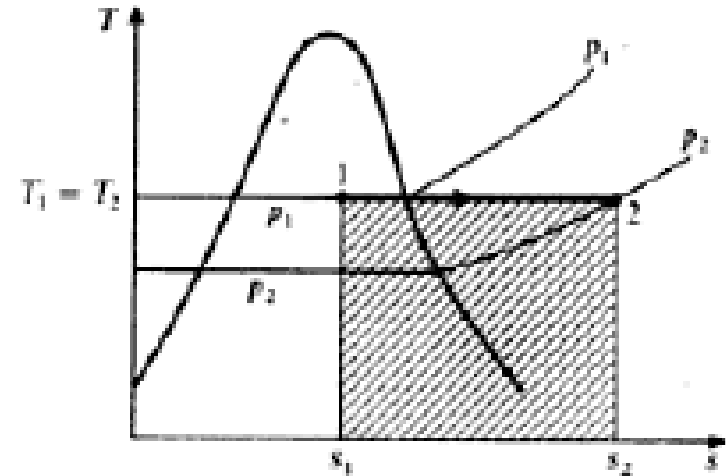


Fig. 4.14: Reversible isothermal process for steam on a T-s diagram

The shaded area represents the heat supplied during the process,

i.e.  $\text{Heat supplied} = T(s_2 - s_1)$

Note that the absolute temperature must be used.

## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### REVERSIBLE ISOTHERMAL PROCESS

- A reversible isothermal process for a perfect gas is shown on a T-s diagram in Fig. 4.16.
- The shaded area represents the heat supplied during the process,

$$\text{i.e.} \quad Q = T(s_2 - s_1)$$

- For a perfect gas undergoing an isothermal process it is possible to evaluate  $s_2 - s_1$ . From the non-flow equation we have, for a reversible process,

$$dQ = du + pdv$$

Also, for a perfect gas from Joule's law  $du = c_v dT$ , i.e.

$$dQ = c_v dT + pdv$$

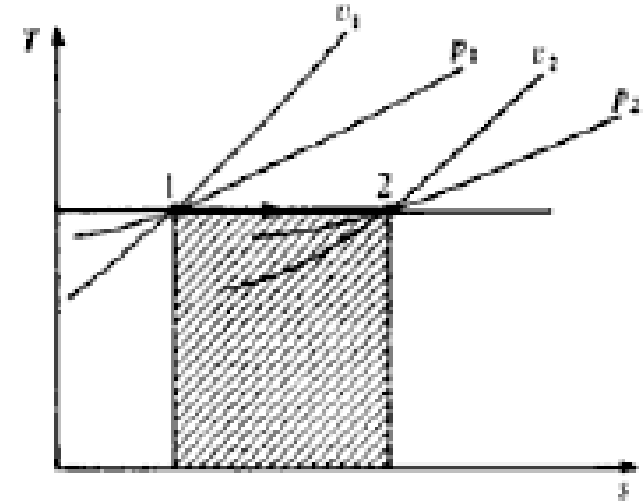


Fig.4.16:Reversible isothermal process for a perfect gas

- For an isothermal process.  $dT = 0$ . hence

$$dQ = p dv$$

- Then, since  $pv = RT$ . we have

$$dQ = RT \frac{dv}{v}$$

- Now from equation (4.8)

$$s_2 - s_1 = \int_1^2 \frac{dQ}{T} = \int_{v_1}^{v_2} \frac{RT dv}{T v} = R \int_{v_1}^{v_2} \frac{dv}{v}$$

## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### REVERSIBLE ISOTHERMAL PROCESS

$$\text{i.e. } s_2 - s_1 = R \ln \left( \frac{v_2}{v_1} \right) = R \ln \left( \frac{p_1}{p_2} \right) \rightarrow (13)$$

Therefore, the heat supplied is given by

$$Q = T(s_2 - s_1) = RT \ln \left( \frac{v_2}{v_1} \right) = RT \ln \left( \frac{p_1}{p_2} \right)$$

Note that this result is the same as that derived before;

$$Q = -W = RT \ln \left( \frac{p_1}{p_2} \right) = p_1 v_1 \ln \left( \frac{p_1}{p_2} \right)$$

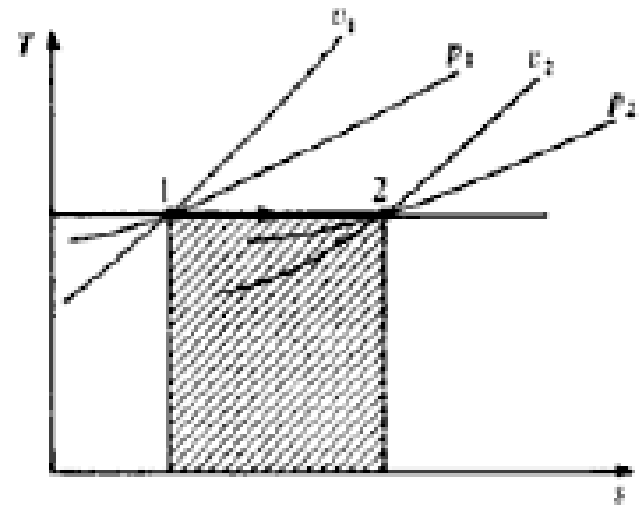
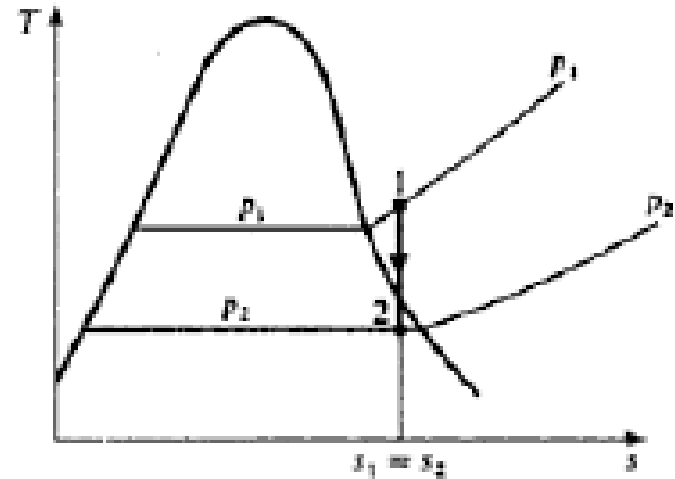


Fig.4.16:Reversible isothermal process for a perfect gas

## **ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM**

### **REVERSIBLE ADIABATIC PROCESS (ISENTROPIC PROCESS)**

- For a reversible adiabatic process, the entropy remains constant, and hence the process is called an isentropic process.
- Note that for a process to be isentropic it need not be either adiabatic or reversible, but the process will always appear as a vertical line on a T-s diagram.
- An isentropic process for superheated steam expanding into the wet region is shown in Fig. 4.18. Now, using the fact that the entropy remains constant, the end states can be found easily from tables.

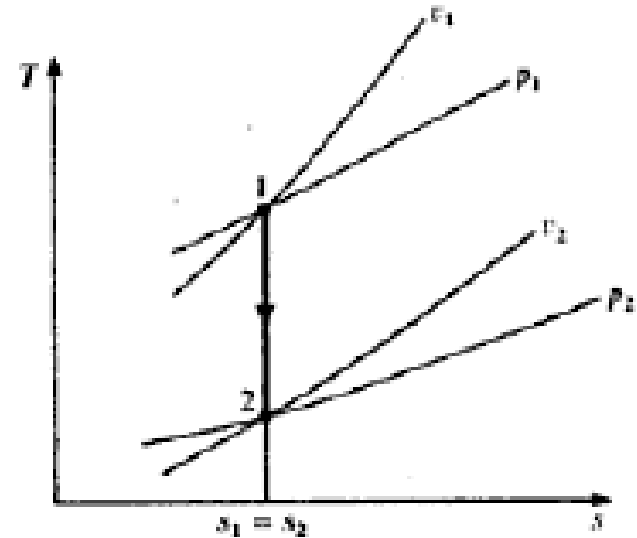


**Fig.4.18: Isentropic process for steam on a T-s diagram**

## **ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM**

### **REVERSIBLE ADIABATIC PROCESS (ISENTROPIC PROCESS)**

- For a perfect gas an isentropic process on a T-s diagram is shown in Fig. 4.19.
- It is shown that for a reversible adiabatic process for a perfect gas the process follows a law  $pv^\gamma = \text{constant}$ .
- Since a reversible adiabatic process occurs at constant entropy and is known as an isentropic process, the index  $\gamma$  is known as the isentropic index of the gas.



**Fig. 4.19: Isentropic process for a perfect gas on a T-s diagram**



## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### POLYTROPIC PROCESS

- To find the change of entropy in a polytropic process for a vapor when the end states have been fixed using

$$P_1 V_1^n = P_2 V_2^n$$

- *Entropy values at the end states can be read straight from tables.*
- The polytropic process is the general case for a perfect gas. To find the entropy change for a perfect gas in the general case, consider the non-flow energy equation for a reversible process.

$$dQ = du + p dv$$

- Also, for unit mass of a perfect gas from Joule's law  $du = c_v dT$  and from equation  $p v = m R T$ .

*Therefore,*

$$dQ = c_v R T + \frac{R T dv}{v}$$

## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### POLYTROPIC PROCESS

Then from equation (4.7)

$$ds = \frac{dQ}{T} = \frac{c_v dT}{T} + \frac{R dv}{v}$$

Hence, between any two states 1 and 2

$$s_2 - s_1 = c_v \int_{T_1}^{T_2} \frac{dT}{T} + R \int_{v_1}^{v_2} \frac{dv}{v} = c_v \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{v_2}{v_1} \right) \rightarrow (4.14)$$

This can be illustrated on a  $T$ - $s$  diagram as in Fig. 4.21.

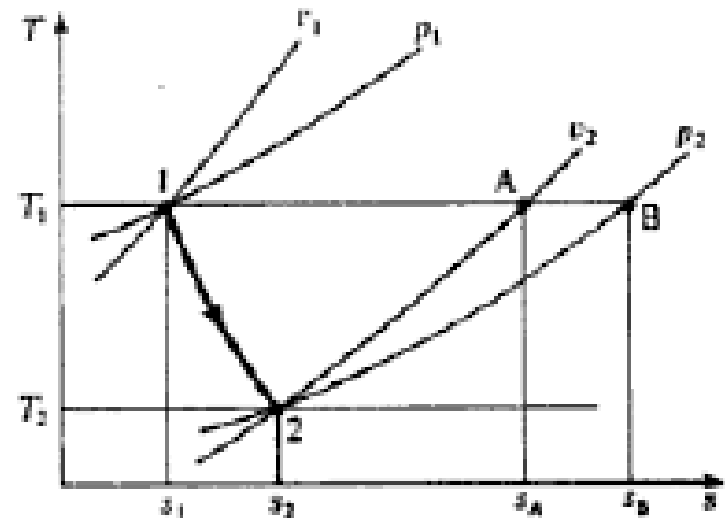


Fig. 4.21 Polytropic process for a perfect gas on a  $T$ - $s$  diagram

## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### POLYTROPIC PROCESS

- Since in the process in Fig. 4.21,  $T_2 < T_1$ , then it is more convenient to write

$$s_2 - s_1 = R \ln \left( \frac{v_2}{v_1} \right) - c_v \ln \left( \frac{T_1}{T_2} \right) \rightarrow (4.15)$$

- The first part of the expression for  $s_2 - s_1$  in equation (4.15) is the change of entropy in an isothermal process from  $v_1$  to  $v_2$ , i.e., from equation (4.13)

$$s_A - s_1 = R \ln \left( \frac{v_2}{v_1} \right)$$

- Also, the second part of the expression for  $s_2 - s_1$ , in equation (4.15) is the change of entropy in a constant volume process from  $T_1$  to  $T_2$ , i.e., referring to Fig. 4.21,

$$s_A - s_1 = c_v \ln \left( \frac{T_1}{T_2} \right)$$

- It can be seen therefore that in calculating the entropy change in a polytropic process from state 1 to state 2 we have in effect replaced the process by two simpler processes: from 1 to A and then from A to 2

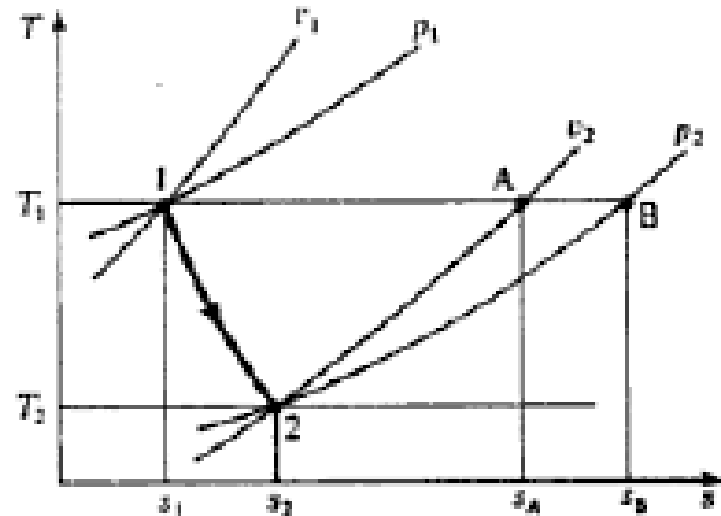


Fig. 4.21 Polytropic process for a perfect gas on a T-s diagram

## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### POLYTROPIC PROCESS

- It is clear from Fig. 4.21 that;

$$s_2 - s_1 = (s_A - s_1) - (s_A - s_2)$$

- Any two processes can be chosen to replace a polytropic process in order to find the entropy change.
- For example, going from 1 to B and then from B to 2 as in Fig. 2.21, we have

$$s_2 - s_1 = (s_B - s_1) - (s_B - s_2)$$

At constant temperature between  $p_1$  and  $p_2$ . using equation (4.13),

$$s_B - s_1 = R \ln \left( \frac{p_1}{p_2} \right)$$

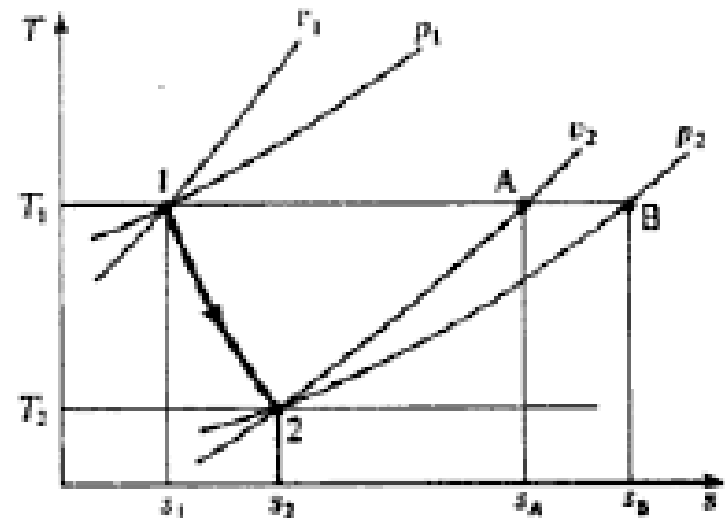


Fig. 4.21 Polytropic process for a perfect gas on a T-s diagram

## ENTROPY-REVERSIBLE PROCESSES ON TS DIAGRAM

### POLYTROPIC PROCESS

- And at constant pressure between  $T_1$  and  $T_2$  we have

$$s_B - s_2 = c_p \ln \left( \frac{T_1}{T_2} \right)$$

- Hence,

$$s_2 - s_1 = R \ln \left( \frac{p_1}{p_2} \right) - c_p \ln \left( \frac{T_1}{T_2} \right)$$

or,

$$s_2 - s_1 = c_p \ln \left( \frac{T_2}{T_1} \right) + R \ln \left( \frac{p_1}{p_2} \right) \rightarrow (4.16)$$

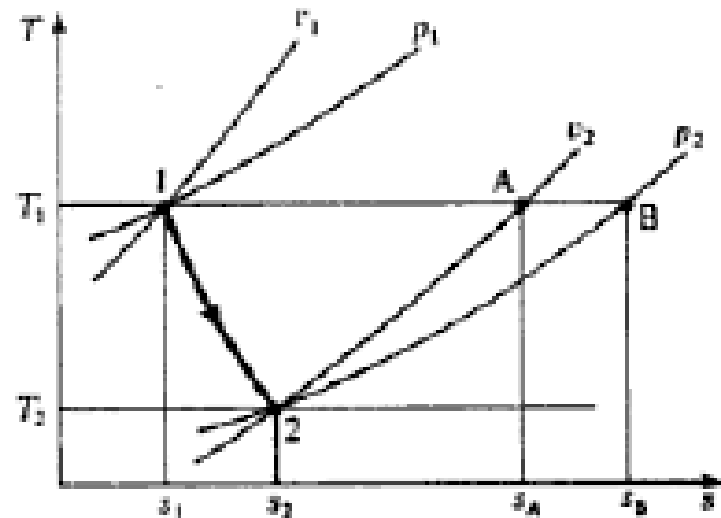


Fig. 4.21 Polytropic process for a perfect gas on a  $T$ - $s$  diagram

- Equation (4.16) can also be derived easily from equation (4.14). There are obviously large number of possible equations for the change of entropy in a polytropic process.
- Each problem can be dealt with by sketching the  $T$ - $s$  diagram and replacing the process by two other simpler reversible processes, as in Fig 4.21.

