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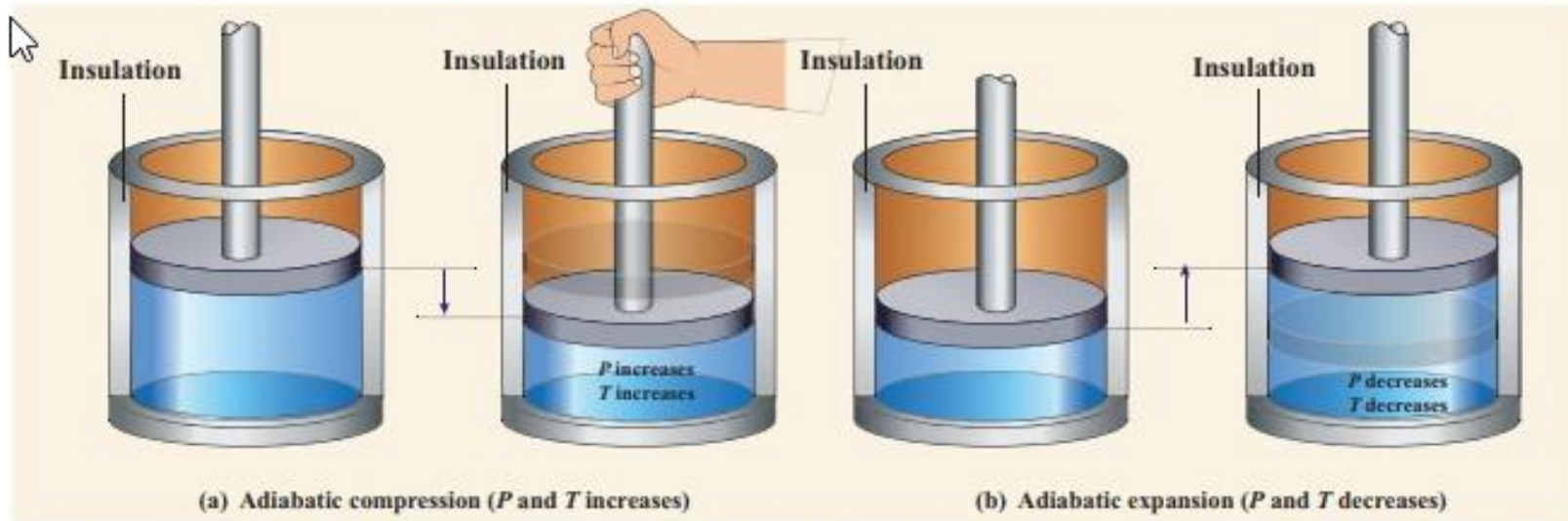
Applied Thermodynamics

Week_09

Instructor: Mr. Adnan Qamar

Mechanical Engineering Department

Reversible and Irreversible Processes
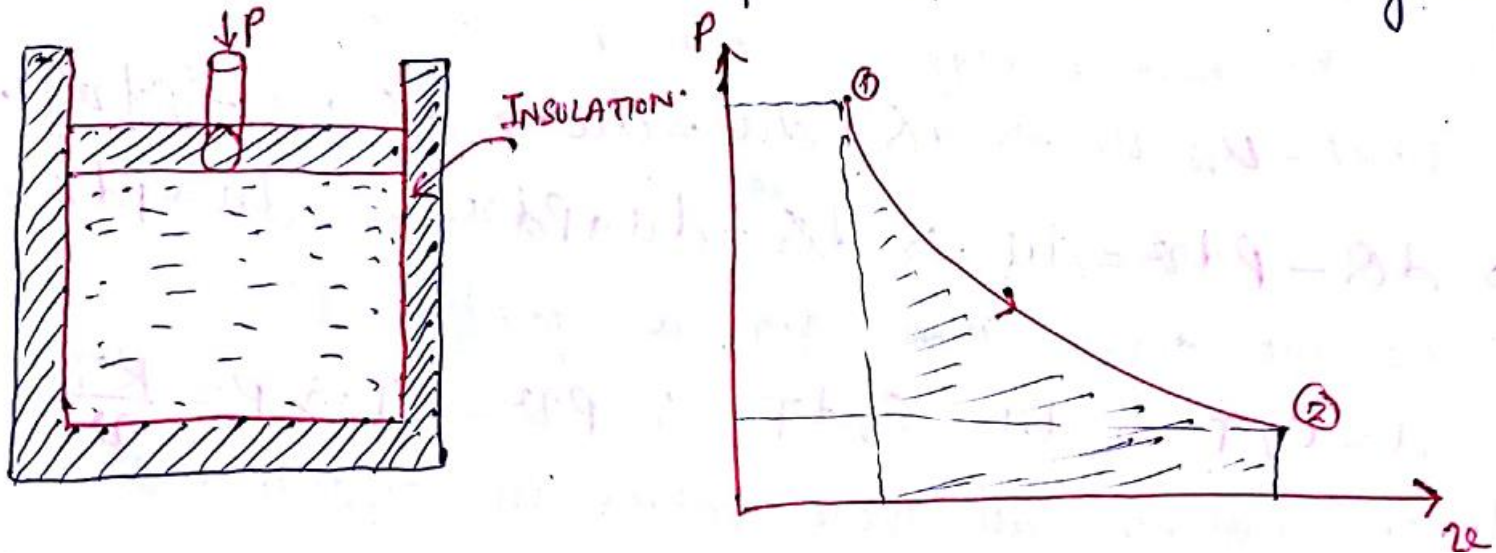
Reversible adiabatic non-flow processes:

➢ An adiabatic process is one in which no heat is transferred to or from the system. Such a process can be reversible or irreversible.

(a) Adiabatic compression (P and T increases)   (b) Adiabatic expansion (P and T decreases)

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

REVERSIBLE ADIABATIC PROCESS FOR VAPOUR: Let us consider a reversible adiabatic non flow Process for a vapour undergoing from state point ① to state point ② as show in Fig. below

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

From non-flow energy equation we have:

$$\cancel{Q}^{0} + W = U_2 - U_1 \Rightarrow W = U_2 - U_1 \longrightarrow ①$$

Equation ① is valid for an adiabatic non flow process whether or not the process is reversible. For a vapour undergoing a reversible adiabatic process the work done can be found by evaluating U_1 and U_2 from steam Tables.

NOTE: In an adiabatic compression process all the work done on the fluid goes to increasing the internal Energy of the fluid, while in an adiabatic expansion process the work done by the fluid is at an expense of reduction in internal Energy of the fluid.

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

REVERSIBLE ADIABATC PROCESS FOR A PERFECT GAS: Let us consider a reversible adabatic process for a perfect gas undergoing from state point ① to State point ② as shown in fig below.
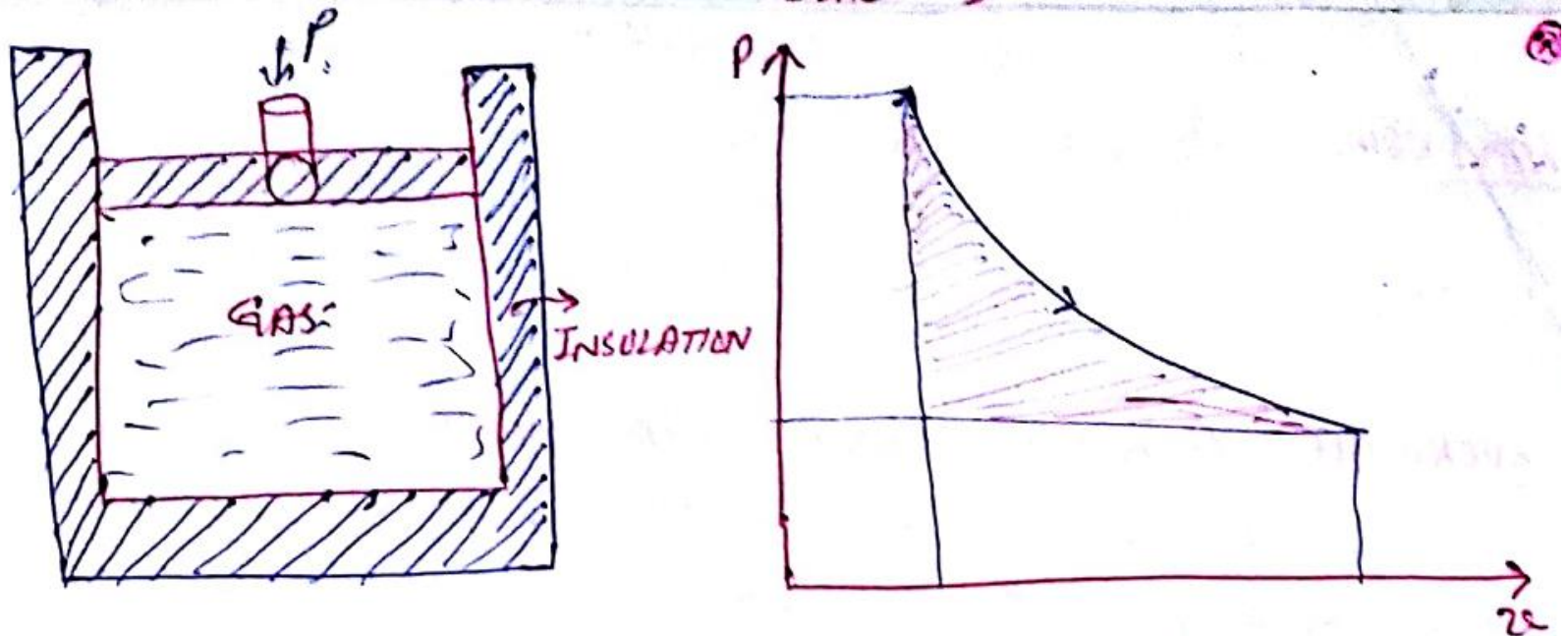
Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

From non flow energy equation we have;

$$Q + W = u_2 - u_1 \Rightarrow dQ + dW = du \Rightarrow dQ + d[-\int P dv] = du.$$

$$\Rightarrow dQ - P dv = du \Rightarrow dQ^{\nearrow 0} = du + P dv \Rightarrow du + P dv = 0 \rightarrow \textcircled{2}$$

Since we know That for a perfect gas;

$$u = C_v T \Rightarrow du = C_v dT \quad ; \quad P v = RT \Rightarrow P = \frac{RT}{v}$$

Hence Putting all These values in equation $\textcircled{2}$ we have

$$C_v dT + \frac{RT}{v} dv = 0$$

Dividing Through by "T" to give a form That can be integrated;

$$C_v \frac{dT}{T} + \frac{R}{v} dv = 0$$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

By integrating Both sides ~~with reference to "T"~~

$$C_V \ln T + R \ln v = \text{Constant} \Rightarrow C_V \ln \left[\frac{Pv}{R}\right] + R \ln v = \text{Constant}$$

$$\Rightarrow \frac{C_V \ln \left[\frac{Pv}{R}\right]}{C_v} + \frac{R \ln v}{C_v} = \frac{\text{Constant}}{C_v} \Rightarrow \ln \left[\frac{Pv}{R}\right] + \frac{R \ln v}{C_v} = \text{Constant}$$

$$\Rightarrow \ln \left[\frac{Pv}{R}\right] + \frac{R \ln v}{(R / (\gamma - 1))} = \text{Constant} \Rightarrow \ln \left[\frac{Pv}{R}\right] + (\gamma - 1) \ln v = \text{Const.}$$

$$\Rightarrow \ln \left[\frac{Pv}{R}\right] + \ln(v^{\gamma - 1}) = \text{Constant} \Rightarrow \ln \left[\frac{Pv \, v^{\gamma-1}}{R}\right] = \text{Constant}$$

$$\Rightarrow \ln \left[\frac{Pv^{\gamma}}{R}\right] = \text{Constant} \Rightarrow \frac{Pv^{\gamma}}{R} = e^{\text{Constant}} \Rightarrow \frac{Pv^{\gamma}}{R} = \text{Constant}$$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

$$\Rightarrow P v^{\gamma} = R \times Constant \Rightarrow P v^{\gamma} = constant \longrightarrow ③$$

we Therefore have a Simple relationship b/w "P" and "v" for any perfect gas undergoing a reversible adiabatic process. Each perfect gas has its own value of "γ". From eq ③ we have:

$$P_1 v_1^{\gamma} = P_2 v_2^{\gamma} \Rightarrow \frac{P_1}{P_2} = \left(\frac{v_2}{v_1}\right)^{\gamma} \longrightarrow ④$$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

RELATIONSHIP BETWEEN "T" & "v" since we know that from eq.

$$pv = RT \implies P = RT/v$$

By Putting This value in eq ③ we have;

$$P \cdot v^\gamma = \text{Constant} \implies \left[\frac{RT}{v}\right] v^\gamma = \text{constant} \implies Tv^{\gamma-1} = \frac{\text{constant}}{R}$$

$$\implies Tv^{\gamma-1} = \text{constant} \implies T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} \implies \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} \quad\text{——(5)}$$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

RELATIONSHIP BETWEEN "T" AND "P": Since we know That from eq.

$$Pv = RT \implies v = RT/P$$

By Putting This value in eq ③ we have;

$$Pv^\gamma = \text{constant} \implies P\left[\frac{RT}{P}\right]^\gamma = \text{constant} \implies \frac{T^\gamma}{P^{\gamma-1}} = \frac{\text{constant}}{R^\gamma}$$

$$\implies \frac{T^\gamma}{P^{\gamma-1}} = \text{constant} \implies \frac{T_1^\gamma}{P_1^{\gamma-1}} = \frac{T_2^\gamma}{P_2^{\gamma-1}} \implies \frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} \longrightarrow ⑥$$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

From equation ① The work done for an adiabatic process for a perfect gas can be calculated by;

$$W = U_2 - U_1 = C_v (T_2 - T_1) = \left(\frac{R}{\gamma - 1}\right)(T_2 - T_1) = \frac{R(T_2 - T_1)}{\gamma - 1} = W$$

$$\Rightarrow W = \frac{P_2 v_2 - P_1 v_1}{\gamma - 1} \longrightarrow ⑦$$

the above expression is used to Calculate work for a perfect gas.

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

EXAMPLE # 3·4: 1 kg of steam at 100 bar and 375°C ④ expands reversibly in a perfectly Thermally insulated cylinder... behind a pisten untill The pressure is 38 bar and The steam is Then dry saturated. Calculate The work done.

GIVEN DATA:

$m = 1 kg$

$P_1 = 100 bar$

$T_1 = 375°C$

$Q = 0$
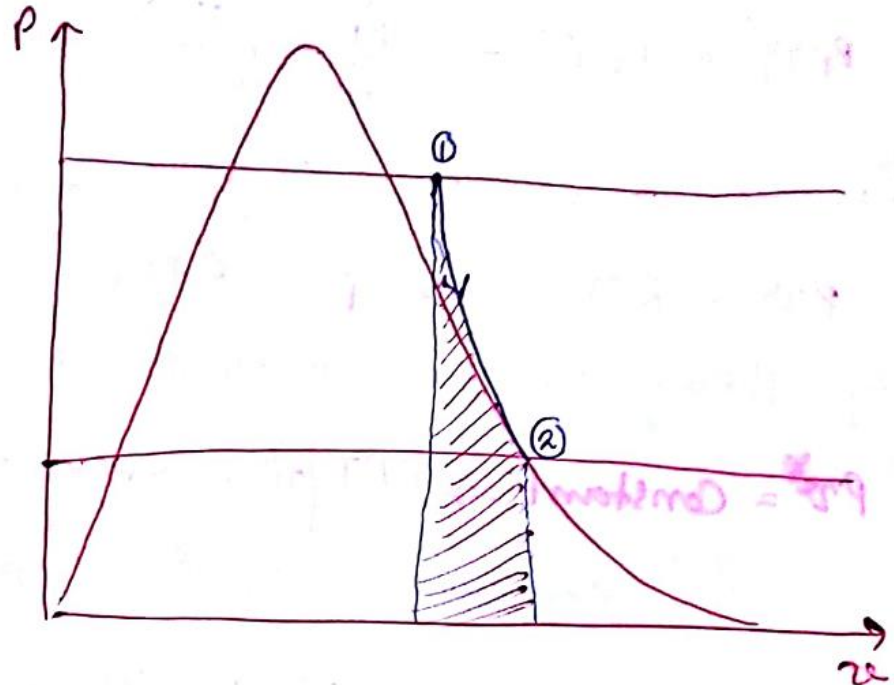
$P_2 = 38 bar$

$x_2 = 1$

$W = ?$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

SOLUTION:

Since we know that for a reversible adiabatic process

$$W = U_2 - U_1 \longrightarrow ①$$

Now from superheated steam Tables;

At $P_1 = 100$ bar ; $T_1 = 375°C$; $h_1 = 3017 \, kJ/kg$; $v_1 = 0.02453 \frac{m^3}{kg}$

$$\Rightarrow \quad U = h - Pv = 3017 - \frac{(100 \times 10^5)(0.02453)}{10^3} = 2771.7 \, kJ/kg$$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

Similarly: At $P_2 = 38$ bar; $x_2 = 1$; $u_2 = u_g = 2602 \, kJ/kg$.

Putting all these values in equation ① we have;

$$W = 2602 - 2771.7 = -169 \, kJ/kg.$$

Work done by the Steam $= +169 \, kJ/kg.$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

Example 3.6

Air at 1.02 bar, 22°C, initially occupying a cylinder volume of 0.015 m³, is compressed reversibly and adiabatically by a piston to a pressure of 6.8 bar. Calculate the final temperature, the final volume, and the work done on the mass of air in the cylinder.
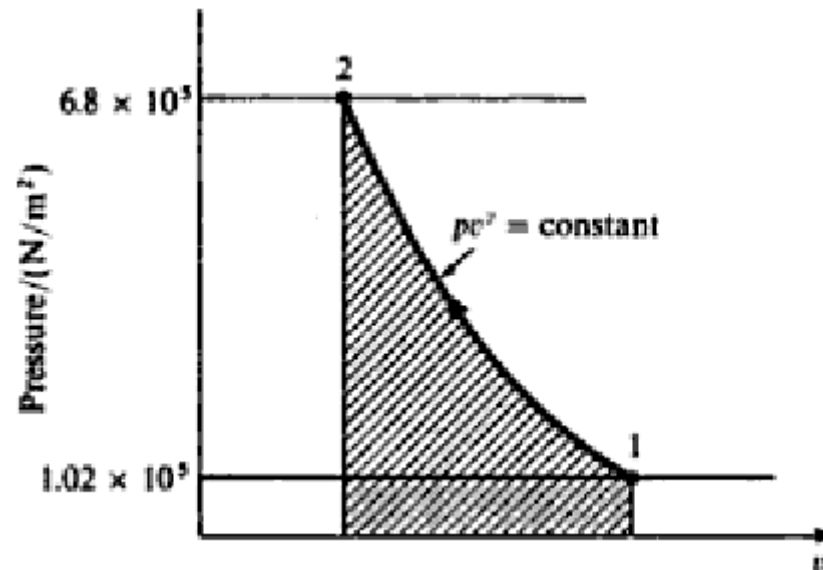
Given Data

$P1 = 1.02$ bar
$T1 = 22°C$
$V1 = 0.015$ m3
$Q = 0$
$P2 = 6.8$ bar
$T2 = ?$
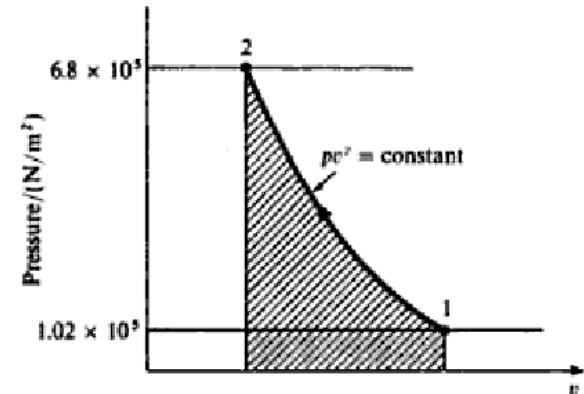$V2 = ?$
$W = ?$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma} \quad \text{or} \quad T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{(\gamma-1)/\gamma}$$

i.e. $\quad T_2 = 295 \times \left(\frac{6.8}{1.02}\right)^{(1.4-1)/1.4}$

$$= 295 \times 1.7195 = 507.3 \text{ K}$$

where $T_1 = 22 + 273 = 295$ K; γ for air $= 1.4$,

i.e. $\quad$ Final temperature $= 507.3 - 273 = 234.3\,°C$

Reversible and Irreversible Processes

Reversible adiabatic non-flow processes:

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} \quad \text{or} \quad \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{1/\gamma}$$
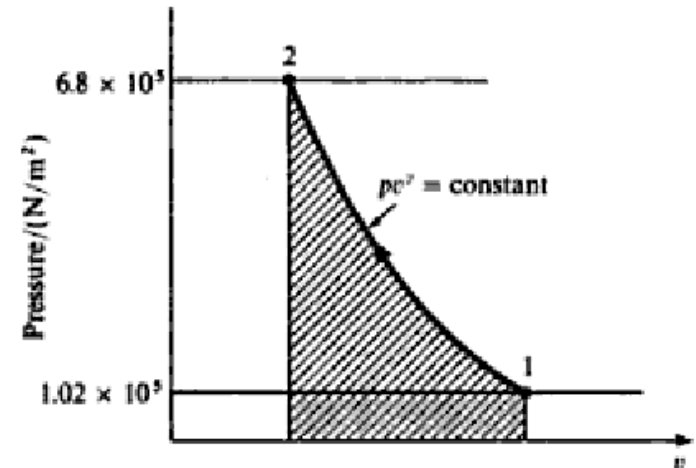
therefore

$$\frac{0.015}{V_2} = \left(\frac{6.8}{1.02}\right)^{1/1.4} = 3.877$$

therefore

$$V_2 = \frac{0.015}{3.877} = 0.003\,87\ \text{m}^3$$

i.e.   Final volume $= 0.003\,87\ \text{m}^3$

Reversible and Irreversible Processes
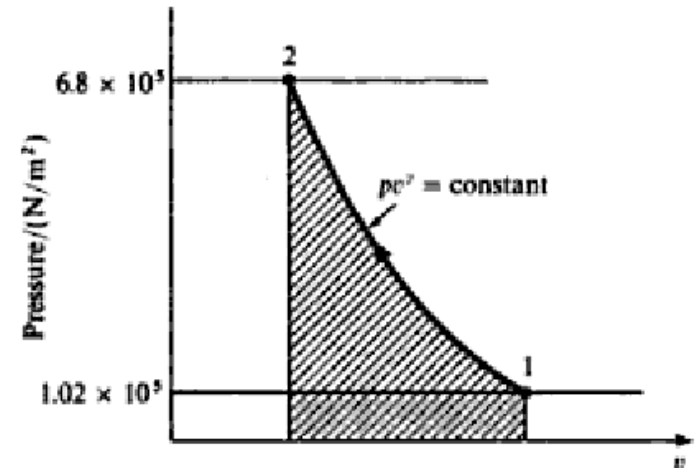
Reversible adiabatic non-flow processes:

$$W = u_2 - u_1$$

$$W = c_v(T_2 - T_1) = 0.718(507.3 - 295)$$

$$= 152.4 \text{ kJ/kg}$$

Work input $= 152.4 \text{ kJ/kg}$

$$m = \frac{p_1 v_1}{R T_1} = \frac{1.02 \times 10^5 \times 0.015}{0.287 \times 10^3 \times 295} = 0.0181 \text{ kg}$$

Total work done $= 0.0181 \times 152.4 = 2.76 \text{ kJ}$

Reversible and Irreversible Processes

Polytropic Process:

➢ A polytropic process is a <u>thermodynamic process</u> that obeys the relation:

$$pv^n = cons\tan t \Rightarrow p = cv^{-n}$$

➢ The exponent *n* may have any value from minus infinity to plus infinity depending on the process. The above correlation can also be written as;

$$p_1 v_1^n = p_2 v_2^n \Rightarrow \frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^n$$

The above equation is true for any working substance undergoing a reversible polytropic process.

Reversible and Irreversible Processes

Polytropic Process for Vapors:

➤ For any reversible non flow process the work done can be expressed as;

$$W = -\int p\,dv = -\int c v^{-n}\,dv$$

$$W = c\,\frac{v_2^{-n+1} - v_1^{-n+1}}{n-1} = \frac{p_2 v_2 v_2^{-n+1} - p_1 v_1 v_1^{-n+1}}{n-1}$$

$$W = \frac{p_2 v_2^{n} v_2^{-n+1} - p_1 v_1^{n} v_1^{-n+1}}{n-1} = \frac{p_2 v_2 - p_1 v_1}{n-1} \rightarrow (1)$$

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

➤ Consider now a polytropic process for a perfect gas;

$$pv^n = cons \tan t \Rightarrow p = cv^{-n}$$

Since $pv = RT \Rightarrow p = \frac{RT}{v}$

$$\Rightarrow \frac{RT}{v} v^n = constant \Rightarrow \frac{T}{v} v^n = \frac{constant}{R} \Rightarrow Tv^{n-1} = contsnat$$

$$\Rightarrow T_1 v_1^{n-1} = T_2 v_2^{n-1} \Rightarrow \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^n$$

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

➢ Relation between Temperature (T) and Pressure (p);

$$pv^n = cons\tan t \Rightarrow p = cv^{-n}$$

Since $\quad pv = RT \Rightarrow v = \dfrac{RT}{p}$

$$\Rightarrow p\left[\frac{RT}{p}\right]^n = constant \Rightarrow \frac{T}{p^{\frac{n-1}{n}}} = \frac{constant}{R^{-n}} \Rightarrow \frac{T}{p^{\frac{n-1}{n}}} = contsnat$$

$$\Rightarrow \frac{T_1}{p_1^{\frac{n-1}{n}}} = \frac{T_2}{p_2^{\frac{n-1}{n}}} \Rightarrow \frac{T_1}{T_2} = \frac{p_1^{\frac{n-1}{n}}}{p_2^{\frac{n-1}{n}}} \Rightarrow \frac{T_1}{T_2} = \left[\frac{p_1}{p_2}\right]^{\frac{n-1}{n}}$$

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

➤ For a perfect gas expanding polytropically it is sometimes more convenient to express the work input in terms of the temperatures at the end states.

$$W = \frac{p_2 v_2 - p_1 v_1}{n-1} = \frac{RT_2 - RT_1}{n-1} = \frac{R(T_2 - T_1)}{n-1}$$

$$W = \frac{P_2 V_2 - P_1 V_1}{n-1} = \frac{mRT_2 - mRT_1}{n-1} = \frac{mR(T_2 - T_1)}{n-1}$$

➤ Using the non-flow energy equation, the heat flow during the process can be found:

$$Q + W = u_2 - u_1 = c_v(T_2 - T_1)$$

$$Q + \frac{R(T_2 - T_1)}{n-1} = c_v(T_2 - T_1)$$

$$Q + \frac{R(T_2 - T_1)}{n-1} = \left[\frac{R}{(\gamma - 1)}\right](T_2 - T_1)$$

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

$$Q + \frac{R(T_2 - T_1)}{n - 1} = \left[\frac{R}{(\gamma - 1)}\right](T_2 - T_1)$$

$$Q = \left[\frac{R}{(\gamma - 1)}\right](T_2 - T_1) - \frac{R(T_2 - T_1)}{n - 1}$$

$$Q = R(T_2 - T_1)\left[\frac{1}{(\gamma - 1)} - \frac{1}{n - 1}\right]$$

$$Q = R(T_2 - T_1)\left[\frac{n - 1 - \gamma + 1}{(\gamma - 1)(n - 1)}\right]$$

$$Q = R(T_2 - T_1)\left[\frac{n - \gamma}{(\gamma - 1)(n - 1)}\right]$$

$$Q = \left[\frac{n - \gamma}{(\gamma - 1)}\right]\frac{R(T_2 - T_1)}{(n - 1)} = \left[\frac{n - \gamma}{(\gamma - 1)}\right]W$$

➤ The above equation is a convenient and concise expression relating the heat supplied and the work input in a polytropic process.

➤ In a compression process work is done on the gas, and hence the term *W is positive. Therefore it can be seen from the equation that when the polytropic index n is greater than γ, in a compression process, then the right-hand side of the equation is positive (i.e. heat is supplied during the process). Conversely, when n is less than γ in a compression process, then heat is rejected by the gas.*

➤ Similarly, the work input in an expansion process is negative, therefore when n is greater than γ, in an expansion process, heat is rejected; and when n is less than γ, in an expansion process, heat must be supplied to the gas during the process. for all perfect gases has a value greater than unity.

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

Example 3.6:

At the commencement of compression in the reciprocating compressor of a refrigeration plant the refrigerant is dry saturated at 1 bar. The compression process follows the law $pv^{1.1}$ = constant until the pressure is 10 bar. Using the properties of refrigerant given in Table interpolating where necessary, calculate:

(i) the work done on the refrigerant during the process;

(ii) the heat transferred to or from the cylinder walls during the process.

Saturation values					Superheat values at 10 bar	
P_s	t_s	v_s	h_f	h_g	v	h
(bar)	(°C)	(m³/kg)	(kJ/kg)		(m³/kg)	(kJ/kg)
1	−30	0.160	8.9	174.2		
10	42	0.018	76.3	203.8	0.020	224.0

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

Given Data:

P1=1 bar X1=1 $Pv^{1.1} = constant$ P2=10 bar   W=?   Q=?

Solution   (i) From Table 3.1, $v_1 = v_{g1} = 0.16$ m³/kg. We then have

$$\Rightarrow \frac{p_1}{p_2} = (\frac{v_2}{v_1})^n \Rightarrow \qquad \frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{1/1.1} = \left(\frac{1}{10}\right)^{0.909} = 0.1233$$

therefore

$$v_2 = 0.1233 \times 0.16 = 0.01973 \text{ m}^3/\text{kg}$$

From equation (3.24)

$$W = \frac{p_2 v_2 - p_1 v_1}{n - 1}$$

$$= \left\{ \frac{(10 \times 0.01973) - (1 \times 0.16)}{1.1 - 1} \right\} \times 10^5$$

$$= 37\,300 \text{ N m} = 37.3 \text{ kJ}$$

i.e.   Work done on the refrigerant = 37.3 kJ

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

(ii) To find the heat transferred it is first necessary to evaluate the internal energies at the end states. Using equation (1.9), $h = u + pv$, we have,

$$u_1 = u_{g1} = h_{g1} - p_{g1} v_{g1}$$

$$= 174.2 - \left(\frac{1 \times 10^5 \times 0.16}{10^3} \right) = 158.2 \text{ kJ/kg}$$

Interpolating from superheat tables at 10 bar, $v_2 = 0.01973 \text{ m}^3/\text{kg}$ we have

$$h_2 = 203.8 + \frac{(0.01973 - 0.018)}{(0.02 - 0.018)} \times (224 - 203.8)$$

$$= 221.3 \text{ kJ/kg}$$

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

Then using equation (1.9)

$$u_2 = 221.3 - \left(\frac{10 \times 10^5 \times 0.01973}{10^3} \right) = 201.6 \text{ kJ/kg}$$

From equation (1.4)

$$Q + W = (u_2 - u_1)$$

therefore

$$Q = -37.3 + (201.6 - 158.2) = 6.1 \text{ kJ/kg}$$

i.e. the heat transferred from the cylinder walls to the refrigerant during the compression process is 6.1 kJ/kg.

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

Example 3.7:

1 kg of a perfect gas is compressed from 1.1 bar, 27 °C according to a law $pv^{1.3}$ = constant, until the pressure is 6.6 bar. Calculate the heat flow to or from the cylinder walls:

(i) When the gas is ethane (molar mass 30 kg/kmol), which has c_p = 2.10 kJ/kg K.

(ii) When the gas is argon (molar mass 40 kg/kmol), which has c_p = 0.520 kJ/kg K.

Given Data:

m=1 kg P1 = 1.1 bar		T1 = 27°C	$Pv^{1.3} = constant$
P2 = 6.6 bar		Q?	
i)	Gas = Ethane	$\tilde{m}$=30 kg/kmol	$c_p = 2.10 \frac{kJ}{kgK}$
ii)	Gas = Argon	$\tilde{m}$=40 kg/kmol	$c_p = 0.520 \; kJ/kgK$

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

Solution   From equation (3.29), for both ethane and argon,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(n-1)/n} \quad \text{or} \quad T_2 = T_1\left(\frac{p_2}{p_1}\right)^{(n-1)/n}$$

i.e. $\quad T_2 = 300\left(\frac{6.6}{1.1}\right)^{1.3-1/1.3} = 300 \times 6^{0.231} = 300 \times 1.512 = 453.6 \text{ K}$

where $T_1 = 27 + 273 = 300$ K.

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

(i) From equation (2.9), $R = \tilde{R}/\tilde{m}$, therefore, for ethane

$$R = \frac{8.3145}{30} = 0.277 \text{ kJ/kg K}$$

Then from equation (2.17), $c_p - c_v = R$, therefore

$$c_v = 2.10 - 0.277 = 1.823 \text{ kJ/kg K}$$

where $c_p = 1.75$ kJ/kg K for ethane. Then from equation (2.20)

$$\gamma = \frac{c_p}{c_v} = \frac{2.10}{1.823} = 1.152$$

From equation (3.30)

$$W = \frac{R(T_2 - T_1)}{n - 1} = \frac{0.277 \times (453.6 - 300)}{1.3 - 1} = 141.8 \text{ kJ/kg}$$

Then from equation (3.32)

$$Q = \left(\frac{n - \gamma}{\gamma - 1}\right)W = \left(\frac{1.3 - 1.152}{1.152 - 1}\right) \times 141.8 = 138.1 \text{ kJ/kg}$$

i.e.   Heat supplied $= 138.1$ kJ/kg

Reversible and Irreversible Processes

Polytropic Process for a Perfect Gas:

(ii) Using the same method for argon we have

$$R = \frac{8.3145}{40} = 0.208 \text{ kJ/kg K}$$

Also $\quad c_v = 0.520 - 0.208 = 0.312 \text{ kJ/kg K}$

therefore

$$\gamma = \frac{0.520}{0.312} = 1.667$$

Then the work input is given by

$$W = \frac{R(T_1 - T_2)}{n - 1} = \frac{0.208 \times (453.6 - 300)}{1.3 - 1} = 106.5 \text{ kJ/kg}$$

Then, $\quad Q = \left(\frac{n - \gamma}{\gamma - 1}\right) W = \left(\frac{1.3 - 1.667}{1.667 - 1}\right) \times 106.5 = -58.6 \text{ kJ/kg}$

i.e. $\quad$ Heat rejected $= 58.6 \text{ kJ/kg}$

Reversible and Irreversible Processes

Reversible Flow Processes:

Although flow processes in practice are usually highly irreversible, it is sometimes convenient to assume that a flow process is reversible in order to provide an ideal comparison. An observer travelling with the flowing fluid would appear to see a change in thermodynamic properties as in a non-flow process. For example, in a reversible adiabatic process for a perfect gas, an observer travelling with the gas would appear to see a process $pv^\gamma =$ constant taking place, but the work input would not be given by $-\int p\,dv$, or by the change in internal energy as given by equation (3.13). Some work is done by virtue of the forces acting between the moving gas and its surroundings. For example, for a reversible adiabatic flow process for a perfect gas, from the flow equation (1.10), for unit mass flow rate

$$\left(h_1 + \frac{C_1^2}{2}\right) + Q + W = \left(h_2 + \frac{C_2^2}{2}\right)$$

Then since $Q = 0$

$$W = (h_2 - h_1) + \left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right)$$

Also, since the process is assumed to be reversible, then for a perfect gas, $pv^\gamma =$ constant. This equation can be used to fix the end states. Note that even if the kinetic energy terms are negligibly small the work input in a reversible adiabatic flow process between two states is not equal to the work input in a reversible adiabatic non-flow process between the same states (given by equation (3.13) as $W = u_2 - u_1$).

Reversible and Irreversible Processes

Reversible Flow Processes:

Although flow processes in practice are usually highly irreversible, it is sometimes convenient to assume that a flow process is reversible in order to provide an ideal comparison. An observer travelling with the flowing fluid would appear to see a change in thermodynamic properties as in a non-flow process. For example, in a reversible adiabatic process for a perfect gas, an observer travelling with the gas would appear to see a process $pv^\gamma = $ constant taking place, but the work input would not be given by $-\int p\, dv$, or by the change in internal energy as given by equation (3.13). Some work is done by virtue of the forces acting between the moving gas and its surroundings. For example, for a reversible adiabatic flow process for a perfect gas, from the flow equation (1.10), for unit mass flow rate

$$\left(h_1 + \frac{C_1^2}{2}\right) + Q + W = \left(h_2 + \frac{C_2^2}{2}\right)$$

Then since $Q = 0$

$$W = (h_2 - h_1) + \left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right)$$

Also, since the process is assumed to be reversible, then for a perfect gas, $pv^\gamma = $ constant. This equation can be used to fix the end states. Note that even if the kinetic energy terms are negligibly small the work input in a reversible adiabatic flow process between two states is not equal to the work input in a reversible adiabatic non-flow process between the same states (given by equation (3.13) as $W = u_2 - u_1$).

Reversible and Irreversible Processes

Reversible Flow Processes :

Example 3.8:

A gas turbine receives gases from the combustion chamber at 7 bar and 650 °C, with a velocity of 9 m/s. The gases leave the turbine at 1 bar with a velocity of 45 m/s. Assuming that the expansion is adiabatic and reversible in the ideal case, calculate the power output per unit mass flow rate. For the gases take $\gamma = 1.333$ and $c_p = 1.11$ kJ/kg K.

Given Data:

P1 = 7.0 bar   T1 = 650°C   C1 = 9 m/s   P2 = 1.0 bar   C2 = 45 m/s

W=? $\gamma = 1.333$ $c_p = 1.11 \frac{kJ}{kgK}$

Reversible and Irreversible Processes

Reversible Flow Processes :

Solution   Using the flow equation for an adiabatic process

$$W = \dot{m}\left\{(h_2 - h_1) + \left(\frac{C_2^2 - C_1^2}{2}\right)\right\}$$

For a perfect gas from equation (2.18), $h = c_p T$, therefore,

$$W = \dot{m}\left\{c_p(T_2 - T_1) + \left(\frac{C_2^2 - C_2^2}{2}\right)\right\}$$

To find T_2 use equation (3.21),

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{(\gamma-1)/\gamma}$$

i.e. $$\frac{T_1}{T_2} = \left(\frac{7}{1}\right)^{(1.333-1)/1.333} = 1.626$$

therefore

$$T_2 = \frac{T_1}{1.626} = \frac{923}{1.626} = 567.7 \text{ K}$$

where $T_1 = 650 + 273 = 923$ K.

Reversible Flow Processes :

Hence substituting for unit mass flow rate

$$\dot{W} = 1 \times 1.11(567.7 - 923) + \left(\frac{45^2 - 9^2}{2 \times 10^3}\right)$$

therefore

$$\dot{W} = -394.4 + 0.97 = -393.4 \text{ kW}$$

i.e.   Power output per kilogram per second $= 393.4 \text{ kW}$

Reversible and Irreversible Processes

Exercise Problems: 3.8, 3.9, 3.10, 3.11, 3.12, 3.13, 3.14, 3.15